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# Risk Analysis 101: Robust-Optimization — the elephant in the robust-satisficing room\*

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## Preface

In 2001, info-gap decision theory re-invented the then 40-year old model of local robustness, known universally as *radius of stability* (circa 1960). Since then, this model of local robustness has been promoted by info-gap scholars as a reliable tool for the management of a severe uncertainty that is characterized by a vast (e.g. unbounded) uncertainty space, a poor point estimate of the uncertainty parameter, and a likelihood-free quantification of uncertainty. Inexplicably, this absurd proposition has managed to pass muster in the review processes of academic books and journals. Small wonder then that info-gap's robustness model was subsequently proposed, in a peer-reviewed article, as a framework for dealing with Taleb's *Black Swans* and even . . . *Unknown Unknowns*?!

More recently, the promotion of info-gap decision theory from the pages of peer-reviewed journals, has been conducted under the more general banner of the great merit of the *robust-satisficing* approach in decision-making, to the effect that advocates of this theory are now engaged in re-inventing the well-established field of *Robust Optimization*.

The trouble in all this is that the misguided rhetoric on *robust-satisficing* coming out of the info-gap literature, specifically the misguided rhetoric on the advantage of *satisficing* over *optimizing* that ends obscuring the obvious connection of robust-satisficing to *Robust Optimization*, had not been recognized for what it is, in the review process of peer-reviewed journals, such as *Risk Analysis*.

In view of this state-of affairs, my objective in this discussion is to make it abundantly clear that the suggestion to use info-gap's *robust-satisficing* approach as a framework for decision-making under severe uncertainty is in fact a step backwards to the early days of *Robust Optimization*, a step that completely ignores the tremendous progress over the past forty years or so, in this highly active area of research.

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Australia, *The Land of the Black Swan*  
September 24, 2012

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\*This article was written for the **Risk Analysis 101 Project** to provide a Second Opinion on an article (Ben-Haim 2012) that was published recently in the journal *Risk Analysis*. See Risk-Analysis-101.moshe-online.com.

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# 1 Introduction

While reading the article *Doing Our Best: Optimization and the Management of Risk* (Ben-Haim 2012), that was published recently in *Risk Analysis*, one is immediately struck by the huge elephant sitting in the room. This elephant attests to the failure on the part of info-gap advocates to recognize the nexus of robust-satisficing to *Robust Optimization*, and to the failure, on the part of *Risk Analysis*, to require that info-gap advocates come clean on this issue.

The same applies to the article *What Makes a Good Decision? Robust Satisficing as a Normative Standard of Rational Decision Making* (Schwartz, Ben-Haim, Dasco 2011), that was published in the *Journal for the Theory of Social Behavior*. Like Ben-Haim (2012), this article also advocates robust-satisficing as the preferred alternative to performance, or utility, maximization. Strangely enough, this article is not even referred to in Ben-Haim (2012).

The reason for my appeal to the elephant in the room metaphor, is to remind the reader of the following hard facts.

*Robust Optimization* is a well established, highly active branch of optimization theory that is concerned with the modeling, analysis, and solution of optimization problems whose solutions are required to be robust against variation in the value of the problem's parameters (Gupta and Rosenhead 1968, Rosenhead et al. 1972, Mulvey et al. 1995, Bai et al. 1997, Kouvelis and Yu 1997, Ben-Tal and Nemirovski 1999, 2002, Bertsimas and Sim 2004, Ben-Tal et al. 2006, Ben-Tal et al. 2009, Bertsimas et al. 2011).

Such variations are often, although not always, due to *uncertainty*. Consequently, *Robust Optimization* is often, although by no means always, a key factor in *decision-making under uncertainty*. This close association between the two endeavors has thus prompted some scholars to describe *Robust Optimization* as being inherently *uncertainty* oriented:

Explicitly considering uncertainty is a critical aspect of decision making. Failure to include uncertainty may lead to very expensive, even disastrous consequences if the anticipated situation is not realized. Thus, it is important to find an optimal (or near optimal) solution that is not overly sensitive to any specific realization of the uncertainty. This is the fundamental goal of robust optimization.

Bai et al. (1997, p. 896)

Robust optimization is an approach for modeling optimization problems under uncertainty, where the modeler aims to find decisions that are optimal for the worst-case realization of the uncertainties within a given set. Typically, the original uncertain optimization problem is converted into an equivalent deterministic form (called the robust counterpart) using strong duality arguments and then solved using standard optimization algorithms. This approach dates back to Soyster (1973), who considered a deterministic linear optimization model that is feasible for all data lying in a convex set.

Goh and Sim (2011, p. 973)

The term “robust optimization” has come to encompass several approaches to protecting the decision-maker against parameter ambiguity and stochastic uncertainty. At a high level, the manager must determine what it means for him to have a robust solution: is it a solution whose feasibility must be guaranteed for any realization of the uncertain parameters? or whose objective value must be guaranteed? or whose distance to optimality must be guaranteed? The main

paradigm relies on worst-case analysis: a solution is evaluated using the realization of the uncertainty that is most unfavorable.

Gabrel et al. (2012)

Another important fact to note about *Robust Optimization* is that, in line with the central role of *constraints* in optimization in general, a key concern in *Robust Optimization* is that of determining *robustness with respect to constraints*. That is, identifying solutions (decisions) that are robust against variations in the value of parameters associated with the *constraints* imposed on the problem.

Traditionally, robust optimization has been used to immunize deterministic optimization problems against infeasibility caused by perturbations in model parameters, while simultaneously preserving computational tractability. The general approach involves reformulation of the original uncertain optimization problem into a deterministic convex program, such that each feasible solution of the new program is feasible for all allowable realizations of the model uncertainties. The deterministic program is therefore “robust” against perturbations in the model parameters.

Goh and Sim (2010, p. 902)

To go back then to the two articles under examination here, the elephant in the room is in fact of such dimensions that *Robust Optimization* scholars would no doubt wonder whether the authors of Schwartz et al. (2011) and Ben-Haim (2012), the referees who reviewed them, and those who accepted them for publication, even heard of the thriving field of *Robust Optimization*.

For, consider what these articles argue: the basic argument in Schwartz et al. (2011) and in Ben-Haim (2012) is that because optimal solutions are often non-robust against the uncertainty in the parameters of the problem, optimization of performance levels, or utility, is an unsuitable approach for the management of uncertainty. A far better approach is one that seeks decisions that are robust against uncertainty with respect to **constraints** imposed on the performance levels.

It is as though the field of *Robust Optimization*, which was established decades ago specifically for the purpose of identifying methodologies seeking **robust optimal solutions**, does not deal precisely with the issue that often optimal solutions to “conventional” optimization problems may not be robust.

Obviously, I wouldn’t have a clue who the reviewers of Schwartz et al. (2011) and Ben-Haim (2012), or who the associate editors who recommended them for publication, were. But what I know for a fact is that the complete lack of reference to *Robust Optimization* in these two articles is not an accident but rather part of a pattern in the info-gap literature (e.g. Ben-Haim 2001, 2006, 2010) to disassociate *robust-satisficing* from *Robust Optimization*.

True, compared to the manner in which robust-satisficing is argued for in *What Makes a Good Decision? Robust Satisficing as a Normative Standard of Rational Decision Making* (Schwartz et al. 2011), the tone in Ben-Haim (2012) is in fact relatively subdued. And to give you an idea of the tone of the former, consider the following extracts from it:

First, we try to specify what counts as “radical uncertainty,” by discussing various approaches to the meaning of statements of probability. Second, we argue that robust satisficing really is a different normative standard for making decisions and not just a prescriptive alternative to utility maximizing that acknowledges human information-processing limitations.

Schwartz et al. (2011, p. 210)

The maximizer of utility seeks the answer to a single question: which option provides the highest subjective expected utility. The robust satisficer answers two questions: first, what will be a “good enough” or satisfactory outcome; and second, of the options that will produce a good enough outcome, which one will do so under the widest range of possible future states of the world.

Schwartz et al. (2011, p. 213)

Robust satisficing is certainly not a description of what decision makers typically do—at least not yet. But is it normative or prescriptive? We believe it is normative.

Schwartz et al. (2011, p. 219)

The foregoing has attempted to make the argument that as a normative matter, robust satisficing is a better strategy for decision making than utility maximizing under conditions of radical uncertainty, and that this is true whether or not the decision space overwhelms the information-processing capacities of the decision maker.

Schwartz et al. (2011, p. 223-224)

Despite the numerous and varied applications of info-gap robust satisficing that we referred to earlier, one rarely, if ever, sees this discussed in the popular literature. Even when risk management and its failures got enormous attention in the aftermath of the financial crisis, all the criticism was of faulty utility maximizing calculation. The possibility that utility maximization was the wrong thing to be calculating was unexplored. It is our hope that in making the normative argument we have here, we will encourage more people to think about robust satisficing as the rational strategy to be following in their own lives, and in the lives of the institutions of which they are a part.

Schwartz et al. (2011, p. 224)

Clearly then, considering the vital role that *Robust Optimization* plays both in optimization theory and in robust decision-making, reading statements such as these, practically compel one to invoke the elephant in the room metaphor. Because, the discussions in Schwartz et al. (2011) and in Ben-Haim (2012) about the proposed *robust-satisficing* approach, not only give no clue of the kinship of *robust-satisficing* to *Robust Optimization*. They effectively give the impression that there is no such field of expertise as *Robust Optimization* and that there are no methodologies that aim at requiring optimal solutions to be robust. I should also note that this is also the case in the three books on info-gap decision theory (Ben-Haim 2001, 2006, 2010), where not a single reference is made to *Robust Optimization*.

As an aside, I should also point out that perhaps even more amazing is the lack of all reference in the book *Info-Gap Economics: An Operational Introduction* (Ben-Haim 2010), to the seminal work by Lars Peter Hansen and Thomas J. Sargent on the use of robustness analysis in economics, notably the lack of all reference to their book *Robustness* (Hansen and Sargent, 2007).

So, for the benefit of readers who are not conversant with this topic, I want to straight-away point out the following:

The *robust-satisficing* approach that is being advanced as info-gap decision theory’s forte (Ben-Haim 2001, 2006, 2010), is no more and no less than a simple, indeed naive, *Robust Optimization* approach. This fact is discussed in considerable detail in Sniedovich (2007, 2008, 2010, 2012, 2012a, 2012b), and elsewhere, where it is explained, and formally proved, that info-gap’s robustness model is no more and no less than a simple instance of Wald’s

maximin model (Wald 1939, 1945, 1950; Luce and Raiffa 1957, Resnik 1987, French 1988) and therefore a simple instance of what is a bread and butter paradigm in decision theory and *Robust Optimization*.

Therefore, the questions that I concentrate on in this discussion are these:

- The misleading rhetoric and demagoguery on *robust-satisficing* in Schwartz et al. (2011) and in Ben-Haim (2012) aside, what is the nature of the relation between the *robust-satisficing* approach proposed by info-gap decision theory and *Robust Optimization*?
- How can scholarly discussions, in peer-reviewed articles, on the purported advantage of *robust-satisficing* models over *utility maximization models* and *performance-optimizing models*, possibly get away with remaining blatantly oblivious to the thriving field of *Robust Optimization* and its rich literature?

In a nutshell:

- The answer to the first question is that the *robust-satisficing* models offered by info-gap decision theory, are, as I pointed out already, simple, some would say naive, *Robust Optimization* models. To be precise, what makes info-gap's robustness model and info-gap's decision model *Robust Optimization models* is the plain fact that these models are based on a (local) *worst-case* approach to variability/uncertainty that is captured by Wald's famous maximin model (circa 1940). As a matter of fact, info-gap's robustness model is a simple instance of a model that is known universally as the *Radius of Stability* model (circa 1960).
- The answer to the second question is that articles such as Schwartz et al. (2011) and Ben-Haim (2012), and books such as Ben-Haim (2001, 2006, 2010) can remain silent on the field of *Robust Optimization* and get away with it, because the revered peer review process is obviously not foolproof.

One of the main points that I make here is this. Accepting the central propositions in Schwartz et al. (2011) and in Ben-Haim (2001, 2006, 2010, 2012) not only will not advance us one step forward. To the contrary, this will take us backward to the late 1960s, to the era predating the recent advances in the field of *Robust Optimization*. To be precise, accepting these propositions will return us to the era of *Robust Optimization* in its infancy, when it was just emerging as a distinct field in optimization theory designed to meet the need for identifying methodologies aimed at finding optimal solutions that are robust against variations in the values of parameters associated with optimization problems.

And the implication of this is, as I explain in this discussion, that the *robust-satisficing* approach advocated by Schwartz et al. (2011) and Ben-Haim (2012) not only **cannot challenge** that which *Robust Optimization* offers the field of robust decision-making, **it is no match** for the methods and techniques provided by *Robust Optimization* for this purpose.

In sum then: as a simple, indeed naive, *Robust Optimization* approach, info-gap's robust-satisficing approach is effectively **subsumed** by *Robust Optimization* as a special case. This means that the discussions in Schwartz et al. (2011) and Ben-Haim (2012) on the proposed robust-satisficing approach, which is conducted in a manner that totally disregards the field of *Robust Optimization*, give the misleading impression that no such field as *Robust Optimization* exists.

Clearly, not only is this position inexplicable, it is inexcusable!

For this reason, I take up again the main points that I had already discussed in detail in Sniedovich (2007, 2008, 2010, 2012, 2012a, 2012b) and elsewhere, with the view to bring

into sharp focus the relation between the robust-satisficing approach advocated by info-gap decision theory and *Robust Optimization*.

Put another way, my objective in this discussion is to give a perspective on the robust-satisficing approach, advocated by info-gap decision theory, from the standpoint of *Robust Optimization*. I am confident that this discussion will benefit not only info-gap scholars, should they bother to read it, but also referees and associate editors of journals, such as *Risk Analysis*, who apparently prove vulnerable to info-gap's rhetoric and demagoguery on its robust-satisficing approach and its role and place in risk analysis and decision-making under severe uncertainty.

## 2 Robust optimization

For the benefit of readers who are not familiar with *Robust Optimization* (Gupta and Rosenhead 1968, Rosenhead et al. 1972, Mulvey et al. 1995, Bai et al. 1997, Kouvelis and Yu 1997, Ben-Tal and Nemirovski 1999, 2002, Bertsimas and Sim 2004, Ben-Tal et al. 2006, Ben-Tal et al. 2009, 2012; Bertsimas et al. 2011), let me point out that this branch of optimization theory concerns itself with developing methodologies aimed at finding *robust optimal solutions*. These are methodologies that seek robust solutions to optimization problems against variations in the values of parameters of the objective function and constraints of optimization problems.

The goal of robust optimization, which has its roots in stochastic optimization, is to produce a solution whose quality will withstand a wide variety of parameter realizations. Robust optimization seeks to mitigate the effects of uncertainty rather than merely anticipating it. Hence, robustness reflects a tendency to hedge against uncertainty, sacrificing some performance in order to avoid excessive volatility[7]. Robust formulations are designed to yield solutions that are less sensitive to model data than classical mathematical programming formulations. Robust programs fall into two broad categories—*solution robust* programs seek to minimize variance in solution optimality, while *model robust* programs aim to decrease variance in feasibility.

Untiedt (2010, p. 197)

As we can clearly see then, because a key concern in optimization theory is the issue of *constraints*, a key concern in *Robust Optimization* is the identification of solutions that are robust to variations in the value of parameters of *constraints* imposed on an optimization problem, namely solutions that are *model robust*. In other words, robust-satisficing is a central issue in *Robust Optimization* (RO).

For instance, consider this extract from the article *Deriving robust counterparts of nonlinear uncertain inequalities*:

The goal of RO is to immunize an optimization problem against uncertain parameters in the problem. Such uncertain parameters may arise as a result of estimation or rounding errors in the parameter values, or due to implementation errors. Therefore, a so-called uncertainty region for the uncertain parameters is defined, and then it is required that the constraints should hold for all parameter values that reside in the uncertain region. For several optimization problems, and for several choices of this uncertainty region, the so-called Robust Counterpart (RC) can be formulated as a tractable optimization problem. For example the robust counterpart for a linear programming problem with polyhedral or ellipsoidal uncertainty regions can be reformulated as a linear programming or

conic quadratic programming problem, respectively.

Ben-Tal et al. (2012, p. 2)

Hence, an extremely important class of *Robust Optimization* problems would be represented by the following abstract, simple model:

**Robust Optimization Model.**

Find a robust solution to the following parametric optimization problem:

$$\max_{x \in X} f(x; u) \tag{1}$$

$$\text{subject to constraints on } (x, u) \in X \times \mathcal{U} \text{ pairs.} \tag{2}$$

Here  $X$  is some set,  $f$  is a real-valued function on  $X$ , and  $u \in \mathcal{U}$  is a parameter whose set of possible/plausible values,  $\mathcal{U}$ , is given. The notation “ $x; u$ ”, as in  $f(x; u)$ , gives notice that  $x$  is a *decision variable* whose value is controlled by the decision-maker, whereas  $u \in \mathcal{U}$  is a parameter whose range of possible/plausible values is  $\mathcal{U}$ .

We refer to  $\mathcal{U}$  as the *parameter space*, or *uncertainty space*, depending on the application under consideration. So, let

$$con(x; u) = \text{list of constraints imposed on the pairs } (x, u) \in X \times \mathcal{U} \text{ in (2).} \tag{3}$$

Roughly, a *robust solution* to the generic parametric optimization problem specified by (1)-(3) is a solution  $x \in X$  that “performs well” with respect to both the objective function  $f$  and the constraints  $con(x; u)$  for a wide range of values of  $u \in \mathcal{U}$ . In contrast, a *fragile* solution is a solution  $x \in X$  that “performs well” only with respect to a very small set of values of  $u \in \mathcal{U}$ .

One can derive various instances of the above abstract *Robust Optimization Model*, depending on the definition one gives to “performs well” and “robust” in the above framework. For example, if one adopts a *worst-case* approach to uncertainty, as *Wald’s maximin decision rule* (Wald 1939, 1945, 1950; Luce and Raiffa 1957, Rawls 1971, Resnik 1987, French 1988) indeed does, then the *robust-counterpart* of the above parametric optimization problem would be formulated as follows:

$$\max_{x \in X} \min_{u \in \mathcal{U}} \{f(x; u) : con(x; u), \forall u \in \mathcal{U}\} . \tag{4}$$

Take note of the two iconic expressions in this generic maximin model. The expression  $\min_{u \in \mathcal{U}}$ , which represents robustness with respect to the objective function  $f$ , and the expression  $\forall u \in \mathcal{U}$ , which represents robustness with respects to the constraints  $con(x; u)$ .

### 3 Robust-satisficing

From a *Robust Optimization* point of view, robust-satisficing models are robust optimization models that seek robustness only with respect to *constraints*. In the context of the abstract *Robust Optimization* model formulated above, robust-satisficing models are instances where the objective function  $f$  is independent of the parameter  $u$ .

For example, consider the following simplification of the maximin model (4), obtained by assuming that the objective function  $f$  is independent of the parameter  $u$ :

$$\max_{x \in X} \min_{u \in \mathcal{U}} \{f(x) : con(x; u), \forall u \in \mathcal{U}\} = \max_{x \in X} \{f(x) : con(x; u), \forall u \in \mathcal{U}\} . \tag{5}$$

Note that the absence of the iconic expression  $\min_{u \in \mathcal{U}}$  indicates that this maximin model does not seek robustness with respect to the objective function  $f$ . It seeks robustness only with respect to the constraints  $con(x; u)$ , hence the retention of the clause  $\forall u \in \mathcal{U}$ .



That said, take note that the robust-satisficing approach advocated by info-gap decision theory (Ben-Haim 2001, 2006, 2010), addresses *local* robustness problems of the following generic form:

**Local Robust-satisficing Model:**

Find a decision  $x \in X$  that is most robust with respect to the performance requirement  $r_c \leq r(x, u)$  against **small deviations/perturbations** in the value of the parameter  $u \in \mathcal{U}$  in a given **nominal value**  $\tilde{u} \in \mathcal{U}$ .

Here  $r$  is a real-valued function on  $X \times \mathcal{U}$  and  $r_c$  is a numeric scalar representing a *critical level of performance*.

The reference to **small deviations/perturbations** gives notice that **small deviations/perturbations** are considered in the **first instance**. This means that larger deviations/perturbations would be considered only if the decision proves resilient to smaller deviations/perturbations in the value of  $\tilde{u}$ . This goes to show that the nominal value  $\tilde{u}$  plays a critical role in info-gap’s robustness analysis, hence that info-gap’s robustness model is a model of *local* robustness *par excellence*.

This is illustrated in Figure 1, where the sets of feasible (acceptable) values of  $u$  associated with two decisions are represented by the shaded areas. The respective smallest perturbations in the value of  $\tilde{u}$  that can cause the decisions to violate the performance constraint  $r_c \leq r(x, u)$ , if increased ever so slightly, are represented by the (red) arrows.

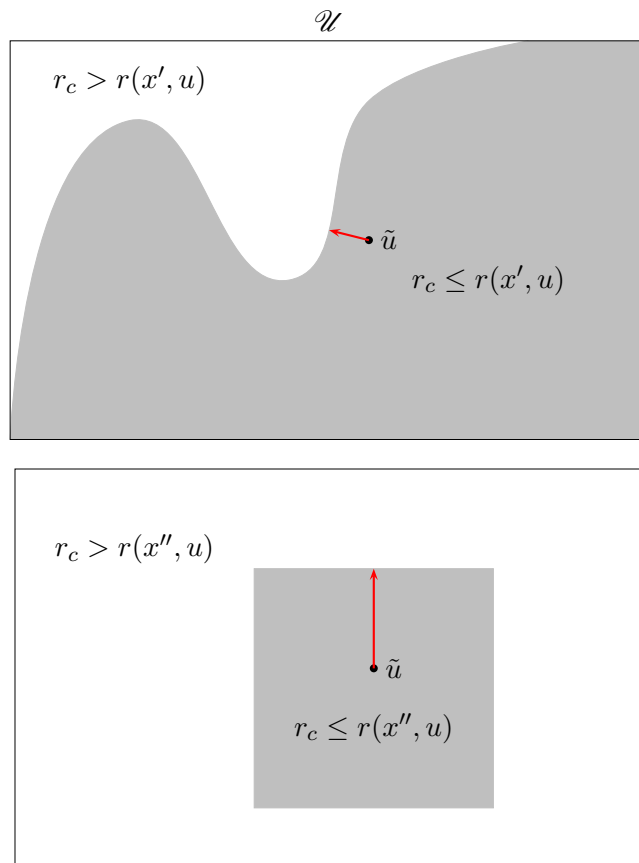


Figure 1: Info-gap robustness of two decisions at  $\tilde{u}$

According to info-gap decision theory, decision  $x''$  is more robust than decision  $x'$  at  $\tilde{u}$  because the critical smallest perturbation associated with decision  $x''$  is larger than the critical smallest perturbation associated with decision  $x'$ . And this in spite of the fact that

decision  $x'$  can withstand very large deviations in  $\tilde{u}$  that decision  $x''$  cannot. This is a manifestation of the inherently *local* orientation of info-gap's robustness model

## 4 Some basic facts

One need not be a *Risk Analyst* to immediately see, indeed this is obvious by inspection, that the *Local Robust-satisficing Model* is significantly narrower in scope than the *Robust Optimization Model*. And what is more, that the latter subsumes the former as a simple, nay naive, special case. To amplify this point, consider this extract from the abstract of the article *Robust optimization of large-scale systems*:

A solution to an optimization model is defined as: *solution robust* if it remains "close" to the optimal for all scenarios of the input data, and *model robust* if it remains "almost" feasible for all data scenario. We then develop a general model formulation, called *robust optimization* (RO), that explicitly incorporates the conflicting objectives of solution and model robustness.

Mulvey et al. (1995, p. 264)

Thus, according to this classification, the *Local Robust-satisficing Model* is a simple *Robust Optimization Model*, simple in that it seeks decisions that are only *model robust*, but not decisions that are both *model robust*, and *solution robust*.

It is also important to point out that, as we shall see, info-gap decision theory offers only one specific measure of *local* robustness, whereas *Robustness Optimization* offers a variety of measures of robustness. And last but not least, info-gap decision theory is not concerned at all with *algorithms* for the robust optimization models generated in the implementation of its robust-satisficing approach. In sharp contrast, the development of *algorithms* for the efficient solution of robust optimization problems is at the very heart of *Robust Optimization* (Kouvelis and Yu 1997, Bertsimas and Sim 2004, Ben-Tal et al. 2009).

To round out this short introductory perspective, I remind the reader of the following facts (see Sniedovich 2007, 2010, 2012, 2012a, 2012b):

- **Fact 1:** Info-gap's robustness model is a re-invention of a staple model of local robustness known universally as *Radius of Stability* (circa 1960), which in turn is a simple instance of Wald's famous *maximin model* (circa 1940). More on this below.
- **Fact 2:** Info-gap decision theory claims to be new and radically different from all current theories of decision under uncertainty.
- **Fact 3:** Info-gap's robustness model is utterly unsuitable for the treatment of the severe uncertainty that is postulated by info-gap decision theory. More on this below.

And to see how grossly misleading the rhetoric in Schwartz et al. (2011) and Ben-Haim (2012) on info-gap's robust-satisficing approach is, it is sufficient to keep in mind the relation between info-gap's robustness model (circa 2000) and these three staple models of robustness:

- Size Criterion model (circa 1968).
- Radius of stability model (circa 1960).
- Wald's maximin model (circa 1940).

The point is that:

- Info-gap's robustness model is a *localized* version of the *Size Criterion* model (circa 1968).

- Info-gap’s robustness model is a simple radius of stability model (circa 1960).
- Info-gap’s robustness model and its robust-satisficing decision model are simple instances of Wald’s maximin model (circa 1940).

Readers of *Risk Analysis* are advised that articles discussing these issues had already been published in this journal (Sniedovich 2012a) and elsewhere (e.g. Sniedovich 2010, 2012, 2012b), which only goes to show how apposite the elephant in the room metaphor is.

## 5 Robustness models

As indicated above, to determine the role and place of info-gap’s robustness model in the state of the art, it is important to relate it formally and concisely to the three robustness models listed above.

### 5.1 Size Criterion

This criterion dates back to the early days of *Robust Optimization* (e.g. Gupta and Rosenhead 1968, Rosenhead et al. 1972). It proposes that the robustness of a decision be measured by the “size” of the set of “satisfactory” values of the parameter of interest. Namely, by the set of values of  $u \in \mathcal{U}$  which, insofar as the decision in question is concerned, “perform well” with regard to pre-specified performance requirements/constraints. Let then

$$A(x) := \{u \in \mathcal{U} : \text{the pair } (x, u) \text{ satisfies the constraints } \text{con}(x; u)\}, \quad x \in X. \quad (6)$$

We refer to  $A(x)$  as the set of *acceptable* values of  $u$  associated with decision  $x$ . Hence, the larger  $A(x)$ , the more (globally) robust  $x$ . The *model robustness* of decision  $x$  would therefore be defined, according to this criterion, as the “size” of  $A(x)$ :

#### Size Robustness:

$$\text{Rob}(x) := \text{size}(A(x)), \quad x \in X \quad (7)$$

$$= \max_{V \subseteq \mathcal{U}} \{\text{size}(V) : \text{con}(x; u), \forall u \in V\} \quad (8)$$

where  $\text{size}(V)$  denotes the “size” of set  $V$  according to a suitable measure of “size”. For instance, if  $V$  is a set consisting of finitely many elements, then we can let  $\text{size}(V) = |V|$  where  $|V|$  denotes the cardinality of set  $V$ .

By definition, the *Size Robustness* of decision  $x$ , denoted  $\text{Rob}(x)$ , is equal to the size of the largest subset of  $\mathcal{U}$  all whose elements are “satisfactory” values of the parameter, namely values of  $u$  that satisfy the conditions specified by  $\text{con}(x; u)$  for decision  $x$ .

Figure 2 illustrates this measure of robustness. The large rectangle represents the parameter space  $\mathcal{U}$ , and the shaded areas represent the sets of acceptable values of  $u$  pertaining to two decisions,  $x'$  and  $x''$ . The *Size Robustness* of a decision would be defined as the size of the shaded area associated with the decision. According to this criterion, decision  $x'$  is much more robust than decision  $x''$ .

It is important to note that, for all its intuitive appeal, this measure of global robustness is of limited practical use because it often proves impossible to formulate  $\text{size}(V)$  in a manner that is amenable to analytic or numerical treatment. There are of course exceptions to this fact (e.g. Starr 1963, 1966; Schneller and Spiccas 1983, Eiselt and Langley 1990, Eiselt et al. 1998, Rosenblat 1987).

As we shall see, info-gap’s robustness model is a “localized” version of this model. That is to say, in the context of info-gap decision theory, set  $V$  in (8) is required to be a

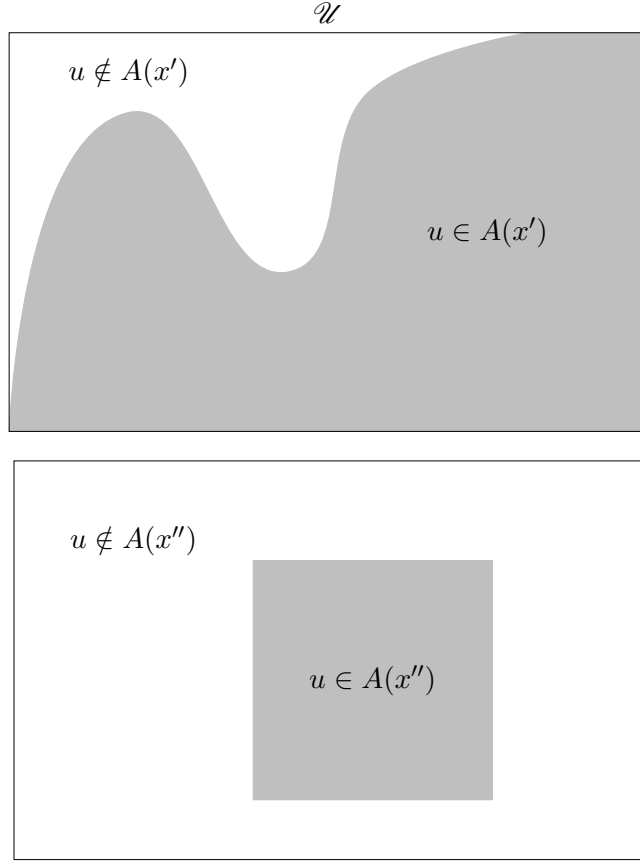


Figure 2: Size Robustness

neighborhood of  $\tilde{u}$ . The trouble is that some info-gap scholars confuse info-gap robustness, which is inherently *local* in nature, with *Size Robustness*, which is a measure of *global* robustness.

## 5.2 Radius of stability model

In very broad terms, the *radius of stability* of a system/decision, at a given nominal value of a parameter of interest, is the size of the **smallest deviation/perturbation** in this nominal value that destabilizes the system/decision. More formally, let  $con(x; u)$  denote the list of conditions (requirements) imposed on decision  $x$  in relation to the parameter  $u \in \mathcal{U}$ . If  $x$  satisfies these conditions at  $u$ , we say that  $x$  is *stable* at  $u$ . Otherwise, it is *unstable* at  $u$ . In this context, the set of acceptable values of  $u$ , namely  $A(x)$ , denotes the set of all the elements of  $\mathcal{U}$  at which decision  $x$  is stable.

The radius of stability of  $x$  at  $u = \tilde{u}$  is a measure of the *local* robustness of  $x$  in the neighborhood of  $\tilde{u}$ . It is defined as follows:

### Radius of Stability Model:

$$\hat{\rho}(x, \tilde{u}) := \min_{\alpha \geq 0} \{ \alpha : U(\alpha', \tilde{u}) \not\subseteq A(x), \forall \alpha' > \alpha \} , \quad x \in X \quad (9)$$

$$= \max_{\alpha \geq 0} \{ \alpha : U(\alpha, \tilde{u}) \subseteq A(x) \} \quad (10)$$

$$= \max_{\alpha \geq 0} \{ \alpha : con(x; u), \forall u \in U(\alpha, \tilde{u}) \} \quad (11)$$

where  $U(\alpha, \tilde{u})$  denotes a neighborhood of size (radius)  $\alpha$  around  $\tilde{u}$ . Such a neighborhood consists of all the elements of  $\mathcal{U}$  that are within a distance  $\alpha$  from  $\tilde{u}$ , according to a suitable measure (metric/norm) of distance on  $\mathcal{U}$ .

This is illustrated in Figure 3, where the radii of stability of two decisions are shown. In this figure the rectangle represents the parameter/uncertainty space  $\mathcal{U}$ , the shaded area represents the set  $A(x)$ , and the circles represent neighborhoods around the nominal point  $\tilde{u}$ . The radius of stability of decision  $x$  is represented by the radius of the largest circle centered at  $\tilde{u}$  that is contained in the shaded area associated with this decision.

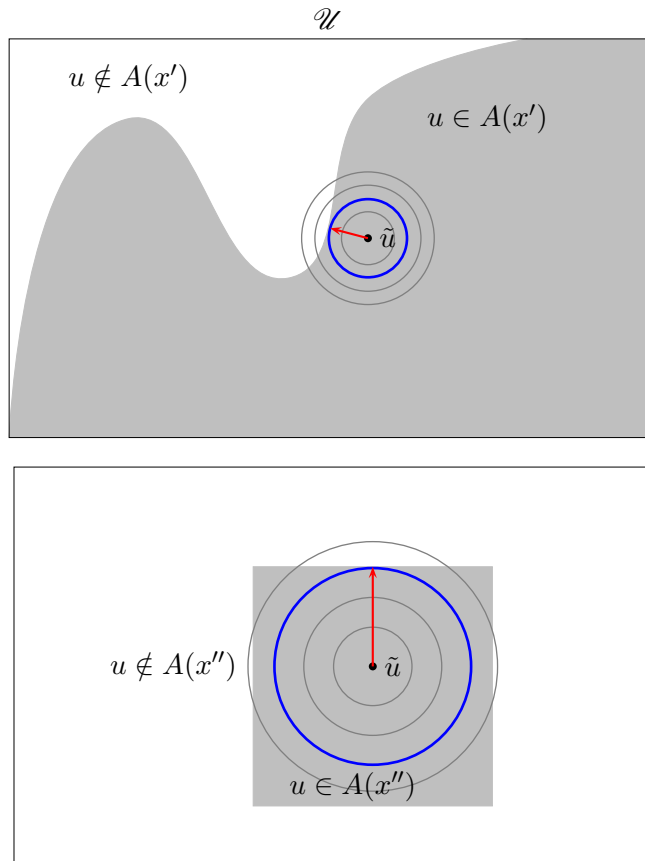


Figure 3: Radii of stability of two decisions at  $\tilde{u}$

Note that although set  $A(x')$  is much larger than set  $A(x'')$ , the radius of stability of decision  $x''$  is much larger than the radius of stability of decision  $x'$ . This illustrates the inherently *local* nature of the radius of stability robustness model.

As we shall see, info-gap's robustness model is a radius of stability model where  $con(x, u)$  consists of the single constraint of the form  $r_c \leq r(x, u)$ .

Radius of stability models are used extensively in many fields, for instance: numerical analysis, applied mathematics, control theory, parametric optimization, operations research, economics, etc. (Wilf 1960, Milne and Reynolds 1962, Hindrichsen and Pritchard 1986a, 1986b; 2005; Paice and Wirth 1998, Zlobec 1988, 2001; Sotskov 1995, 2007; Anderson and Bernfeld 2001, Cooper et al. 2004).

### 5.3 Wald's maximin model

This versatile model and its many variants, for instance the minimax *regret* model (Savage 1951, French 1986, Resnik 1987, Kouvelis and Yu 1997), dominate the scene of non-

probabilistic robustness analysis. They give expression to Wald's (1939, p. 305) observation that if we are in no position to take into account the probability pertaining to the uncertain "state of the world", then it is "reasonable" to assume the *worst*, hence to assume that *the least attractive*, namely *worst*, state of the world will be realized

By the early 1950s, this approach to non-probabilistic uncertainty became the foremost non-probabilistic *decision rule* offered by classical decision theory (Luce and Raiffa 1957, Resnik 1987, French 1988). It is used extensively in *Robust Optimization*.

Before we proceed to examine some relevant mathematical formulations of this rule, consider the following verbal formulation thereof (for a slightly different version see Rawls 1971, p. 152):

**Wald's Maximin Decision Rule:**

Rank alternatives according to their worst outcomes. Hence, select the alternative whose worst outcome is at least as good as the worst outcome of all other alternatives.

As this formulation brings out, the versatility of this *Rule* lies in the fact that the terms "alternative", "outcome", and "worst" are indeterminate, which means that they can be given a specific definition, according to need, to suit the requirements of particular applications of the *Rule*. To illustrate this point, let us consider first the classic "text-book" case where robustness is sought only with respect to the objective function of an optimization problem. So let

$Y =$  set of alternatives.

$S(y) =$  set of states associated with alternative  $y$ .

$g(y; s) =$  outcome generated by alternative  $y$  and state  $s$ ,

where  $g$  is a real-valued function on  $Y$  that depends on the state variable  $s$ . Assume that the decision maker is interested in *maximizing* the outcome  $g(y; s)$ , keeping in mind that the value of  $s$  is "uncertain" and is thus outside the decision maker's control.

In this case, the *worst outcome* associated with alternative  $y$  is determined by *minimizing* the value of  $g(y; s)$  over  $s \in S(y)$ , namely

$$SL(y) := \min_{s \in S(y)} g(y; s), \quad y \in Y. \quad (12)$$

We regard  $SL(y)$  as the *security level* of alternative  $y$ , which as instructed by the *Rule*, is employed to rank the alternatives  $y \in Y$  under consideration. Hence, the best alternative is obtained by solving the following maximin problem:

$$SL^* := \max_{y \in Y} SL(y) \quad (13)$$

$$= \max_{y \in Y} \min_{s \in S(y)} g(y; s). \quad (14)$$

The iconic expression  $\min_{s \in S(y)}$  indicates that a worst-case robustness is sought with respect to the objective function  $g$  against variations in the value of the state  $s$  over  $S(y)$ .

In situations where robustness is also sought with respect to *constraints*, an "outcome" consists of two parts, one representing the value of  $g(y; s)$  and one indicating whether the constraints are satisfied by the  $(y, s)$  pairs. As usual, a *lexicographic priority* is given to constraint satisfaction, hence according to the worst-case approach, alternative  $y$  is considered "admissible" only if it satisfies the constraints for **all**  $s \in S(y)$ . In other words, any alternative  $y \in Y$  such that a pair  $(y, s)$  violates the constraint for some  $s \in S(y)$  is discarded as being inadmissible. The remaining, namely admissible, alternatives, are ranked according to their worst value of  $g(y; s)$  over  $s \in S(y)$ .

If we let  $Con(y; s)$  denote the constraints imposed on the  $(y, s)$  pairs, then the worst-case robustness constraint on alternative  $y$  is as follows:

$$Con(y; s), \forall s \in S(y) . \quad (15)$$

We therefore let

$$sl(y) := \min_{s \in S(y)} \{g(y; s) : Con(y; s), \forall s \in S(y)\} , \quad y \in Y \quad (16)$$

define the *security level* of alternative  $y$  in this case.

It follows therefore that according to the *Rule*, the best alternative is obtained by solving the following maximin model:

$$sl^* := \max_{y \in Y} sl(y) \quad (17)$$

$$= \max_{y \in Y} \min_{s \in S(y)} \{g(y; s) : Con(y; s), \forall s \in S(y)\} . \quad (18)$$

This model makes it crystal clear that a worst-case robustness is sought here both with respect to the objective function  $g$  **and** the constraints  $Con(y; s)$ . The former is manifested in the iconic expression  $\min_{s \in S(y)}$  that is applied to  $g(y; s)$ , and the latter in the iconic expression  $\forall s \in S(y)$  that is applied to the constraints  $Con(y; s)$ .

I need hardly point out that because the robust-satisficing approach advocated by info-gap decision theory is concerned only with robustness with respect to constraints, our overwhelming interest in this discussion is in simple cases of (18) where the objective function  $g$  is independent of the state variable  $s$ . Observe that in this case, the iconic  $\min_{s \in S(y)}$  operation is superfluous, hence the generic maximin model (18) is simplified to

$$sl^\circ := \max_{y \in Y} \{g(y) : Con(y; s), \forall s \in S(y)\} . \quad (19)$$

Maximin models of this type, namely maximin models where the objective function is independent of the state variable  $s$ , are called *MP models* (short for *Mathematical Programming Models*). They are used extensively in *Robust Optimization* because in the absence of the inner min operation, they have the air of “conventional” optimization (maximization) problems (e.g. Ecker and Kupferschmid 1988, pp. 24-25; Kouvelis and Yu, 1997, p. 27). Note, however, that in cases where the sets  $S(y), y \in Y$  consist of infinitely many elements, the optimization problem in question is classified as a *semi-infinite* problem due to the fact that it involves infinitely many constraints (one list  $Con(y; s)$  for every value of  $s \in S(y)$ ).

Now, consider the case where  $y \equiv \alpha, s \equiv u, Y = [0, \infty), S(y) = U(\alpha, \tilde{u})$  and  $Con(y; s) = con(x; u)$  for a given decision  $x \in X$ . In this case, the maximin model (19) takes this form

$$z^*(x) := \max_{\alpha \geq 0} \{g(\alpha) : con(x; u), \forall u \in U(\alpha, \tilde{u})\} , \quad x \in X . \quad (20)$$

It follows therefore that the radius of stability model (11) is that instance of this maximin model which corresponds to  $g(\alpha) \equiv \alpha$ . The implication therefore is that info-gap’s robustness model is a simple instance of this model which corresponds to the simple case where  $g(\alpha) \equiv \alpha$  and the list of constraints  $con(x; u)$  consists of the single constraint  $r_c \leq r(x, u)$ .

## Remark

It is important to take note that the three formats of the maximin model discussed above, namely the classic “text-book” format (14), the generic maximin model (18), and the MP

model (19), are *equivalent*. That is to say, each format represents one and the same generic maximin paradigm, which means that any one of these formats can be easily transformed into an equivalent instance of the other two formats.

And to illustrate, observe that the generic maximin model (18) is equivalent to the following two (equivalent) models:

$$\max_{y \in Y, v \in \mathbb{R}} \{v : v \leq g(y; s), \text{Con}(y; s), \forall s \in S(y)\} \equiv \max_{y \in Y} \min_{s \in S(y)} G(y; s) \quad (21)$$

where

$$G(y; s) := \begin{cases} g(y; s) & , \text{ the pair } (y; s) \text{ satisfies } \text{Con}(y; s) \\ -\infty & , \text{ otherwise.} \end{cases} \quad (22)$$

The large penalty  $(-\infty)$  imposed by  $G$  for violating the constraints  $\text{Con}(y; s)$  is a reflection of the lexicographic preference relation that gives priority to constraint satisfaction over better values of the objective function.

In practice, the choice between the above three equivalent formats of the *Maximin Decision Rule* is a problem oriented modeling issue that often boils down to mathematical convenience.

To illustrate how the maximin paradigm copes with cases with no objective function, consider the following robust-satisficing problem:

**Robust-satisficing problem:**

Find a decision  $x \in X$  such that the constraints listed in  $\text{con}(x; u)$  are satisfied for all  $u \in V$ , for a given set  $V \subseteq \mathcal{U}$ .

In this case, an “outcome” is an indication of whether the decision (alternative) satisfies the constraints for the given value of  $u$ . It goes without saying that our preference indeed is for decisions that satisfy the constraints. This preference can therefore be stipulated by the objective function  $F$  defined on  $X$  as follows:

$$F(x; u) := \begin{cases} 1 & , \text{ the pair } (x; u) \text{ satisfies } \text{con}(x; u) \\ 0 & , \text{ otherwise} \end{cases} \quad , \quad x \in X, u \in \mathcal{U} . \quad (23)$$

The worst outcome associated with decision  $x$  is then the worst (smallest) value of  $F(x; u)$  over  $u \in V$ , in which case the corresponding maximin model is as follows:

$$z^* := \max_{x \in X} \min_{u \in V} F(x; u) . \quad (24)$$

Obviously,  $z^*$  can take only two values, namely 0 or 1. If  $z^* = 0$ , then the conclusion is that the robust-satisficing problem has no solution: there is no  $x \in X$  such that  $\text{con}(x; u), \forall u \in V$ . If, on the other hand,  $z^* = 1$ , then any  $x^* \in X$  such that  $(x^*, u^*)$  is an optimal solution to the maximin problem (24) for some  $u^* \in V$ , is a solution to the robust-satisficing problem, namely  $x^* \in X$  and it satisfies the constraint  $\text{con}(x^*; u), \forall u \in V$ . Note that for any such  $x^*$  we have  $F(x^*, u) = 1, \forall u \in V$ .

Alternatively, we can simply let  $F$  be independent of  $x$  and  $u$ , say set  $F(x) = 1, \forall x \in X$ , and incorporate the constraints  $\text{con}(x; u)$  explicitly in the maximin model as follows:

$$z^\circ := \max_{x \in X} \min_{u \in V} \{F(x) : \text{con}(x; u), \forall u \in V\} \quad (25)$$

$$= \max_{x \in X} \{F(x) : \text{con}(x; u), \forall u \in V\} . \quad (26)$$

Clearly, a decision  $x \in X$  is an optimal solution to this optimization problem iff it is a solution to the above *Robust-satisficing Problem*. It is important to note, though, that in practice, one would often be able to choose far more informative and effective objective functions for robust-satisficing problems of the type considered here.



## 5.4 Info-gap's robustness model

The generic form of info-gap's robustness model (Ben-Haim 2001, 2006) is as follows:

### Info-gap Robustness Model:

$$\hat{\alpha}(x, r_c) := \max_{\alpha \geq 0} \{ \alpha : r_c \leq r(x, u), \forall u \in U(\alpha, \tilde{u}) \}, \quad x \in X. \quad (27)$$

In words, the info-gap robustness of decision  $x$  at  $\tilde{u}$ , denoted  $\hat{\alpha}(x, r_c)$ , is equal to the size ( $\alpha$ ) of the largest neighborhood  $U(\alpha, \tilde{u})$  around  $\tilde{u}$  all whose elements satisfy the constraint  $r_c \leq r(x, u)$ . In this model  $\tilde{u}$  represents a point *estimate* of the true value of  $u$ .

It is assumed that  $U(0, \tilde{u}) = \{\tilde{u}\}$  and that  $\alpha' < \alpha''$  implies  $U(\alpha', \tilde{u}) \subseteq U(\alpha'', \tilde{u})$ . Also, with no loss of generality we assume that  $r_c \leq r(x, \tilde{u}), \forall x \in X$ .

And to see more clearly why this model is concerned, first and foremost, with **small deviations/perturbations** in the value of  $\tilde{u}$ , note that

$$\hat{\alpha}(x, r_c) := \max_{\alpha \geq 0} \{ \alpha : r_c \leq r(x, u), \forall u \in U(\alpha, \tilde{u}) \} \quad (28)$$

$$= \min_{\alpha \geq 0} \{ \alpha : U(\alpha', \tilde{u}) \not\subseteq A(x), \forall \alpha' > \alpha \} \quad (29)$$

$$= \min_{\alpha \geq 0} \{ \alpha : r_c > r(x, u'), u' \in U(\alpha', \tilde{u}), \forall \alpha' > \alpha \} \quad (30)$$

where  $A(x) = \{u \in \mathcal{U} : r_c \leq r(x, u)\}$ .

In words, the info-gap robustness of decision  $x$  at  $\tilde{u}$  is equal to the size of the **smallest deviation/perturbation** ( $\alpha$ ) such that for any larger deviation/perturbation,  $\alpha' > \alpha$ , there is a  $u' \in U(\alpha', \tilde{u})$  that **violates** the performance constraint  $r_c \leq r(x, u)$ .

As is immediately clear, indeed by inspection, this model is a radius of stability model (11) with only one constraint of the form  $r_c \leq r(x, u)$ .

The inherently *local* nature of this model is manifested, for instance, in the fact that even if decision  $x$  performs superbly well almost everywhere on  $\mathcal{U}$ , info-gap decision theory deems this decision fragile if it violates the performance constraint anywhere in a small neighborhood around  $\tilde{u}$ . In other words, info-gap's robustness is a measure of *local* robustness *par excellence*: it represents the robustness of decision  $x$  in the neighborhood of the point estimate  $\tilde{u}$ .

This is illustrated in Figure 4 where the info-gap robustness of two decisions,  $x'$  and  $x''$ , are shown for two point estimates  $\tilde{u}_1$  and  $\tilde{u}_2$ .

Note the following:

- The neighborhood around the point estimate  $\tilde{u}$  specified by the value of  $\hat{\alpha}(x, r_c)$ , namely  $U(\hat{\alpha}(x, r_c), \tilde{u})$ , may not be a good approximation of the set of acceptable values of  $u$ , namely  $A(x')$ . These neighborhoods are represented by the (blue) bold circles.
- As indicated by a comparison of (a) and (b), although decision  $x'$  is much more robust globally than decision  $x''$ , the info-gap robustness of decision  $x'$  is much smaller than the info-gap robustness of decision  $x''$  at  $\tilde{u}_1$ .
- As indicated by a comparison of (a) and (c), as well as (b) and (d), the info-gap robustness of a decision may vary significantly as the value of the point estimate  $\tilde{u}$  varies.

To reiterate then, info-gap's robustness model is a model of local robustness, meaning that the info-gap robustness of a decision is a measure of the robustness of the decision in

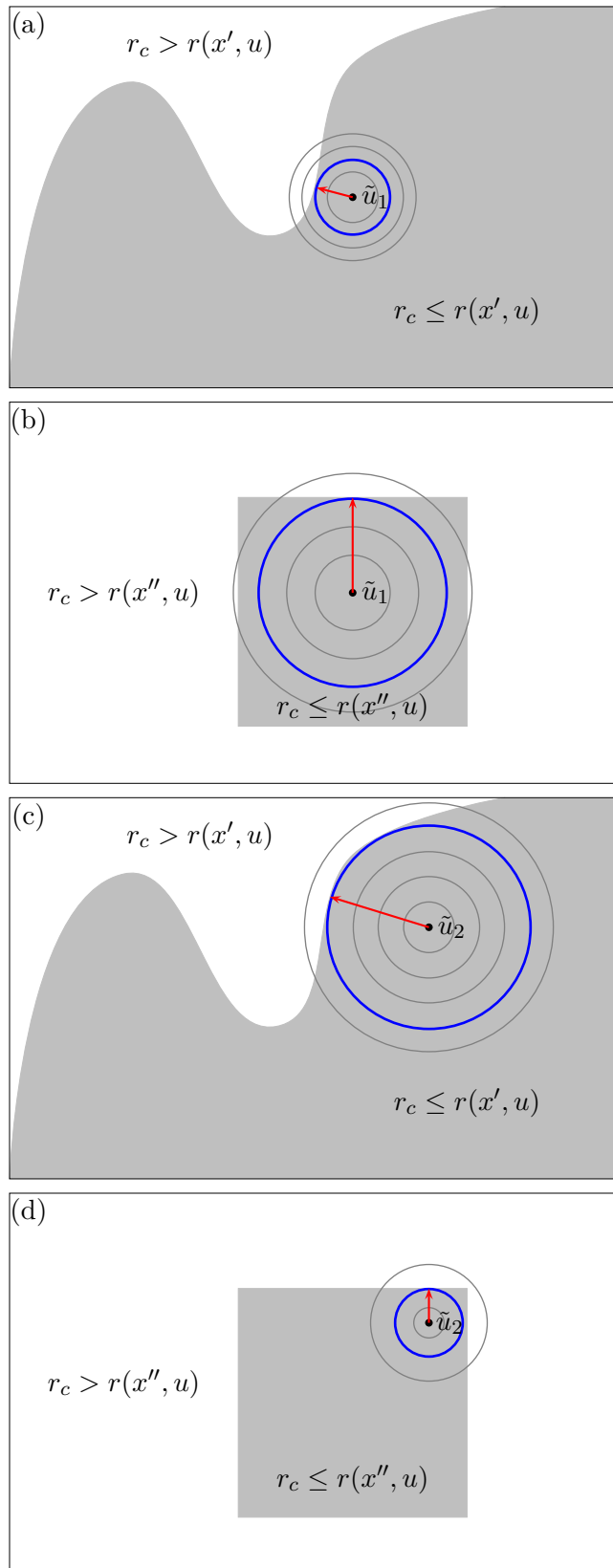


Figure 4: Info-gap robustness of two decisions at two point estimates

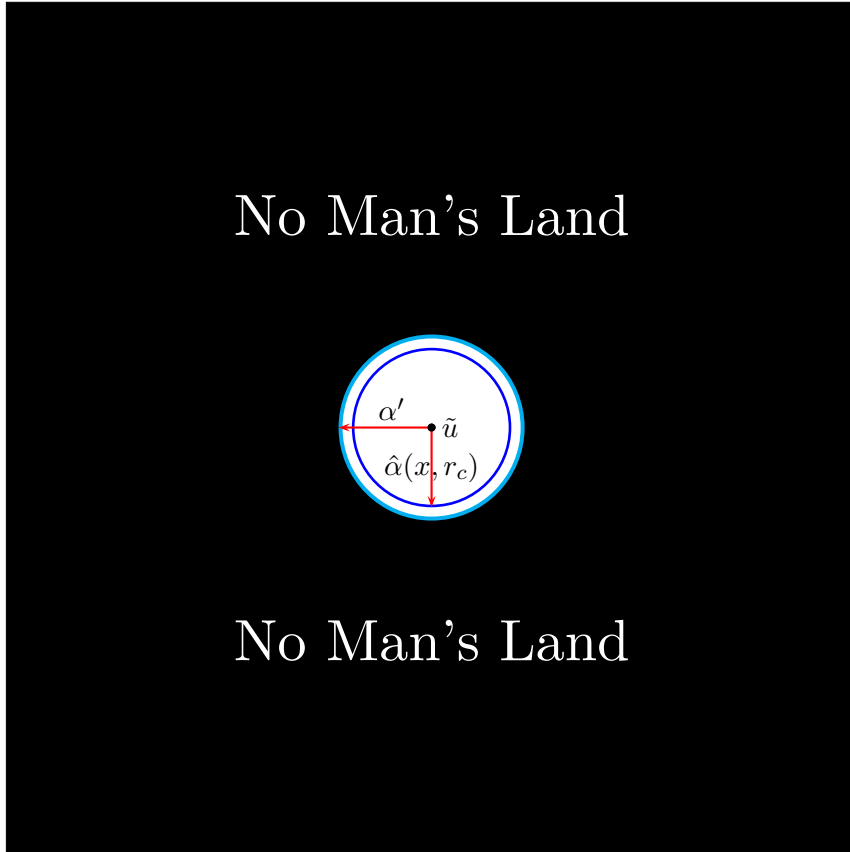


Figure 5: The No Man's Land of info-gap's robustness model

the neighborhood of the point estimate  $\tilde{u}$ . Therefore, methodologically speaking, this model is unsuitable for determining the global robustness of decisions over the uncertainty space  $\mathcal{U}$ . The pathologic case is that where  $x$  satisfies the performance constraint  $r_c \leq r(x, u)$  everywhere in  $\mathcal{U}$ , except at  $\tilde{u}$ . According to info-gap decision theory, the info-gap robustness of this decision is equal to 0.

To put it bluntly:

**Theorem 5.1** *No Man's Land Theorem*

*The info-gap robustness of decision  $x$ , namely the value of  $\hat{\alpha}(x, r_c)$ , is utterly aloof to the performance levels  $r(x, u)$  associated with values of  $u \in \mathcal{U}$  outside the neighborhood  $U(\alpha', \tilde{u})$ , for any  $\alpha'$  that is greater than  $\hat{\alpha}(x, r_c)$ .*

In greater detail: info-gap's robustness model does not even attempt to determine how well decision  $x$  performs vis-a-vis the performance constraint  $r_c \leq r(x, u)$  over the uncertainty space  $\mathcal{U}$ . Indeed, this model ignores completely the performance of decision  $x$  outside the neighborhood  $U(\alpha', \tilde{u})$ , where  $\alpha'$  is slightly greater than  $\hat{\alpha}(x, r_c)$ . The inevitable conclusion therefore is that there is no guarantee that the info-gap robustness of decision  $x$  is a good measure of the robustness of decision  $x$  against the variations in the value of  $u$  over  $\mathcal{U}$ .

This is illustrated in Figure 5, where the black area represents the *No Man's Land* associated with decision  $x$ : this is the set of values of  $u \in \mathcal{U}$  outside the neighborhood  $U(\alpha', \tilde{u})$ , for some  $\alpha'$  that is slightly greater than  $\hat{\alpha}(x, r_c)$ .

## Remark

I remind the reader to keep firmly in mind that the above analysis and its results are about a decision theory that is proclaimed to be a theory for decision-making under severe uncertainty, where the severity of the uncertainty is characterized by:

- A vast (e.g. unbounded) uncertainty space  $\mathcal{U}$ .
- A poor point estimate  $\tilde{u}$ : poor meaning that it can even be a wild guess.
- A likelihood-free quantification of uncertainty.

So, as I explain in Sniedovich (2010, 2012, 2012a, 2012b), and as I indicate below, advocating the use of such a model of local robustness, under such conditions, amounts to advocating *voodoo* decision-making.

## 6 Robust-satisficing info-gap-style

According to the robust-satisficing approach advocated by info-gap decision theory, the more robust a decision, the better. The implication therefore is that this approach seeks to identify the most robust decision(s):

Satisficing means doing well enough, or obtaining an adequate outcome. A *satisficing decision strategy* seeks a decision whose outcome is good enough, though perhaps sub-optimal. A *robust-satisficing decision strategy* maximizes the robustness to uncertainty and satisfies the outcome.

Schwartz et al. (2011, p. 213)

We argue that in decisions under uncertainty, what should be optimized is robustness rather than performance.

Ben-Haim (2012, p. 1)

And what all this comes down to is that to seek out the best decision, the robust-satisficing approach advocated by info-gap decision theory prescribes solving the following optimization problem:

### Local Robust-satisficing Decision Model:

$$\hat{\alpha}(r_c) := \max_{x \in X} \hat{\alpha}(x, r_c) \tag{31}$$

$$= \max_{x \in X} \max_{\alpha \geq 0} \{ \alpha : r_c \leq r(x, u), \forall u \in U(\alpha, \tilde{u}) \} \tag{32}$$

$$= \max_{x \in X, \alpha \geq 0} \{ \alpha : r_c \leq r(x, u), \forall u \in U(\alpha, \tilde{u}) \} . \tag{33}$$

The point to note here is that the iconic expression  $\forall u \in U(\alpha, \tilde{u})$  indicates that a (local) worst-case robustness is sought with respect to the constraint under consideration, while the absence of the corresponding iconic expression  $\min_{u \in U(\alpha, \tilde{u})}$  indicates that robustness is **not** sought with respect to the objective function.

The conclusion is therefore clear as daylight: this model is a maximin model that seeks local robustness with respect to the constraint  $r_c \leq r(x, u)$ . Indeed, by inspection, this model is the instance of (19) that is specified by  $y \equiv (x, \alpha)$ ,  $s \equiv u$ ,  $Y = X \times [0, \infty)$ ,  $S(y) = U(\alpha, \tilde{u})$ ,  $g(y) = \alpha$ , and where  $Con(y; s)$  consists of the single constraint  $r_c \leq r(x, u)$ .

## 7 Where then is the elephant?

You don't have to be a risk analyst to (conclude, as this follows by inspection, that:

**Theorem 7.1** *Info-gap's robustness model (27) is a simple instance of the radius of stability model (11), corresponding to the case where the list of constraints  $\text{con}(x; u)$  consists of the single constraint  $r_c \leq r(x, u)$ .*

**Theorem 7.2** *Info-gap's robustness model (27) is a simple instance of the maximin model (20), corresponding to the case where the list of constraints  $\text{con}(x; u)$  consists of the single constraint  $r_c \leq r(x, u)$  and the objective function is of the form  $g(\alpha) \equiv \alpha$ .*

**Theorem 7.3** *The Local Robust-satisficing Decision Model (33) is a simple instance of the maximin model (19) specified by  $y \equiv (x, \alpha)$ ,  $s \equiv u$ ,  $Y = X \times [0, \infty)$ ,  $S(y) = U(\alpha, \tilde{u})$ ,  $g(y) = \alpha$ , and  $\text{Con}(y; s)$  consisting of the single constraint  $r_c \leq r(x, u)$ .*

**Theorem 7.4** *Info-gap's robustness model (27) is a "localized" version of the Size Robustness model (8), where set  $V$  is required to be a neighborhoods  $U(\alpha, \tilde{u})$ ,  $\alpha \geq 0$  around  $\tilde{u}$ , and the list of constraints  $\text{con}(x; u)$  consists of the single constraint  $r_c \leq r(x, u)$ .*

In a nutshell, by advocating the use of info-gap's robust-satisficing approach, Schwartz et al. (2011) and Ben-Haim (2012) in effect call upon us to go back to the future of the 1950s (in the case of Wald's maximin model), or to the future of the early 1960s (in the case of the radius of stability model), and in so doing to ignore the progress in robust decision-making over the past 50-60 years.

Before I follow this argument to its final conclusion, I want to take up two important issues that require special attention.

## 8 The info-gap rhetoric

As I pointed out at the outset, scores of unsubstantiated, unfounded, groundless and down-right erroneous claims about info-gap decision theory and its robust-satisficing approach have passed muster in the review process of peer-reviewed journals (see Sniedovich 2007, 2010, 2012, 2012a, 2012b). The reason for this state of affairs is simple. The misleading rhetoric pervading the writing on info-gap decision theory and its robust-satisficing approach obscures the hard facts about this theory.

And to illustrate, as I show above, info-gap's robustness model is no more and no less than a simple, more accurately simplistic, indeed naive, reinvented version of a staple model of local robustness, the *radius of stability* model (circa 1960). This model has been used extensively for at least five decades in many fields of expertise. So, as can be expected, this model's mode of operation, its capabilities, hence its scope of operation, are well-understood by numerous scholars in a wide range of disciplines who use this model for the purpose it was designed for, namely to determine **local robustness/stability**.

Yet, the rhetoric in the info-gap literature attributes to this model capabilities that it does not have. Thus, the robust-satisficing approach advocated by info-gap decision theory, which, I need hardly remind the reader, is based on this model of local robustness, is claimed to seek a decision that yields satisfactory outcomes under the **widest range** of conditions/contingencies or states of the world. For instance,

The maximizer of utility seeks the answer to a single question: which option provides the highest subjective expected utility. The robust satisficer answers two questions: first, what will be a "good enough" or satisfactory outcome; and

second, of the options that will produce a good enough outcome, which one will do so under the widest range of possible future states of the world.

Schwartz et al. (2011, p. 213)

This rhetoric **incorrectly** attributes info-gap’s robustness model (27) a capability that is characteristic of the *Size Robustness* model (8). The point of course is that, by virtue of its definition, info-gap’s robust-satisficing model (33) does not seek a decision that performs satisfactorily under the widest range of possible future states of the world. Rather, based as it is on a model of local robustness (27), info-gap’s robust-satisficing model seeks a decision that performs satisfactorily over the largest *neighborhood*  $U(\alpha, \tilde{u})$  around the nominal point  $\tilde{u}$ . This is vividly brought out in Figure 4.

The same applies to this statement from the article *Robust climate policies under uncertainty: a comparison of info-gap and RDM methods*<sup>1</sup>:

The analysis of a continuum of uncertainty from local to global is one of the novel ways in which info-gap analysis is informative.

Hall et al. (2012, p. 6)

The trouble with such misrepresentation of info-gap’s robustness model is that not only do they seem to mislead referees of peer-reviewed journals. They seem to entrench absurd ideas, in info-gap circles, about the capabilities of info-gap’s robustness model, such as its being a reliable tool for the management of a severe uncertainty manifested in extreme events such as seawalls, massive tsunamis, catastrophes and so on (Ben-Haim 2012) and even ... *Black Swans* and *Unknown unknowns* (Wintle et al. 2010).

For an excellent illustration of where info-gap’s misleading rhetoric might land you, consider Sims’ (2001) warning in *Pitfalls of a Minimax Approach to Model Uncertainty* about an unguarded use of minimax models of local robustness in macroeconomics:

They may also—and this is more likely in the recent implementations in macroeconomics—focus the minimaxing on a narrow, technically convenient, uncontroversial range of deviations from a central model. Then the results will remain close to those of the central model, and the danger is that one will be misled by the rhetoric of robustness into devoting less attention than one should to technically inconvenient, controversial deviations from the central model.

Sims (2001, p. 52)

Another illustration of the consequences of being misled by the rhetoric on info-gap’s robustness model is the danger of using models that befit, what Ben-Tal et al. (2009a, p. 926) term ‘... somewhat “irresponsible” decision makers...’, namely decision makers who confine their robustness analysis to the “normal” range of values of the uncertainty parameter thus ignoring “abnormal” values of this parameter.

As I have been arguing all along, advocating the use of such models of local robustness in situations where the uncertainty space is **unbounded** amounts to advocating *voodoo* decision-making (Sniedovich 2010, 2012, 2012a, 2012b).

And to conclude this section, I call attention to another fallacy that has been promulgated by info-gap’s misleading rhetoric: the claim (in various guises) that info-gap’s robustness model is **not** a maximin model. Thus, consider the following assertions:

In a sense info gap analysis may be thought of as extended and structured sensitivity analysis of preference orderings between options. While there is a superficial similarity with minimax decision making, no fixed bounds are imposed on the set of possibilities, leading to a comprehensive search of the set

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<sup>1</sup>See my analysis of this article at <http://www.moshe-online.com/Risk-Analysis-101/>

of possibilities and construction of functions that describe the results of that search.

Hine and Hall (2010, p. 16-17)

The difference from min-max approaches is that we are able to select a policy without ever specifying how wrong the model actually is. Min-max and info-gap robust-satisficing strategies will sometimes agree and sometimes differ.

Ben-Haim (2010, p. 10)

Minimax is a kind of cousin to robust satisficing, but it is not the same. First, at least sometimes, you can't even specify what the worst possible outcome can bring. In such situations, a minimax strategy is unhelpful. Second, and more important, robust satisficing is a way to manage uncertainty, not a way to manage bad outcomes.

Schwartz et al. (2011, p. 222)

The claim that “maximin” is not the same as info-gap’s “robust-satisficing” is, of course, a blatant misrepresentation. For, how could info-gap’s “robust-satisficing” possibly be “the same as maximin” when it is subsumed as a simple case by the maximin paradigm?

The facts speak for themselves: the maximin paradigm is incomparably more general and powerful than the robust-satisficing paradigm advocated by info-gap decision theory. For the record, as indicated by Theorem 7.2, the family ties between info-gap’s robust-satisficing model and Wald’s maximin model are of a different order altogether. Maximin is not a cousin to robust-satisficing. Rather, as a simple instance of Wald’s maximin model, info-gap’s robustness model is a kind of grandchild, and a re-invented “clone” of another grandchild, namely a “clone” of the radius of stability model (circa 1960).

## 9 Satisficing vs optimizing

The argument advanced in the info-gap literature (e.g. Ben-Haim 2001, 2006, 2010) to explain/justify the “satisficing” orientation of info-gap’s measure of robustness, is that, under uncertainty, it is better to “satisfice” than to “optimize”. This is manifested, for instance, in the title of the lecture *Why More is Less: Info-Gap Explanation for Robust-Satisficing Behavior*<sup>2</sup>. Similar catch phrases such as “good is better than best” and “advantage of sub-optimal models” are used for the same purpose.

The argument itself is claimed to be based on Simon’s concept of *Bounded Rationality*:

In summary, info-gap theory provides a quantitative tool for policy formulation and evaluation which is based on Knight’s uncertainty and Simon’s bounded rationality. We cannot predict surprises, but we info-gap theory enables us to model and manage our ignorance of those surprises. Info-gap policy analysis is particularly suited to situations in which surprises are critically important.

Ben-Haim (2010, p. 11)

But, to see how misguided the rhetoric on robust-satisficing, in the info-gap literature, actually is, let us take a quick look at Simon’s bounded rationality.

According to Herbert A. Simon (1916-2001), the Father of *bounded rationality*, the main difficulties facing individuals engaged in a process that aims to determine a “perfectly rational” decision are these:

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<sup>2</sup>See <http://maths.dur.ac.uk/stats/people/fc/13July07.html>

The task of decision involves three steps: (1) the listing of all alternative strategies; (2) the determination of all the consequences that follow upon each of these strategies; (3) the comparative evaluation of these sets of consequences. The word “all” is used advisedly. It is obviously impossible for the individual to know *all* their consequences, and this impossibility is a very important departure of actual behaviour from the model of objective rationality.

Simon (1975, p. 67)

Clearly, accomplishing this task is beyond the capabilities of most individuals (decision-makers):

It has already been remarked that the subject, in order to perform with perfect rationality in this scheme, would have to have a complete description of the consequences following each alternative strategy and would have to compare these consequences. He would have to know in every single respect how the world would be changed by his behaving one way instead of another, and he would have to follow the consequences of behavior through unlimited stretches of time, unlimited reaches of space, and unlimited sets of values.

Simon (1975, p. 69)

From this point of view, *optimization models* can be too detached from the complexities of decision-making in the real-world. So, to take a more realistic stance to real-world decision under uncertainty, Simon proposed to adopt a “satisficing”, rather than an “optimizing”, approach to decision-making:

The central concern of administrative theory is with the boundary between the rational and the non-rational aspects of human social behaviour. Administrative theory is peculiarly the theory of intended and bounded rationality—of the behaviour of human beings who satisfice because they do not have the wits to maximise.

Simon (1976, p. xxviii)

But, it is important to note that Simon did not argue that one should not optimize—when one can—or that satisficing is superior to optimizing. What he did do is to call attention to a rather banal “fact of life” that, in practice, “perfectly rational” optimization is often out of reach.

Although he has occasionally been misunderstood on this, Simon does not a priori discard the optimizing model of choice.

Mongin (2000, p. 74)

On the other hand, while claiming to be rooted in Simon’s pioneering work on *bounded rationality*, info-gap decision theory’s rhetoric is adamant that, under uncertainty, “satisficing” is “better” than “optimizing” period!

And what is so comical in all this is that, for all the fuss that is made about the superiority of satisficing, the robust-satisficing approach advocated in Schwartz et al. (2011) and Ben-Haim (2006, 2010, 2012) in fact prescribes the **maximization** of robustness. The implication therefore is that this approach should be classified as an “optimizing” approach par excellence.

But more than this, to give the reader an immediate sense of the merit of discoursing in the abstract, in general terms, on the advantage of *satisficing* over *optimizing*, I call attention to the fact that it is elementary to show that any satisficing problem can be easily transformed into an equivalent optimization problem. In other words, it is elementary to



prove formally that any satisficing problem can be easily transformed into an optimization problem whose optimal solutions are solutions to the satisficing problem. Here is a quick illustration of such a proof.

Consider the following general, abstract problem:

**Satisficing Problem:**

Find a decision  $x \in X$  that satisfies a given list of constraints.

Next, define the real-valued function  $H$  on  $X$  as follows:

$$H(x) := \begin{cases} 1 & , \quad x \text{ satisfies the constraints} \\ 0 & , \quad \text{otherwise} \end{cases} , \quad x \in X . \quad (34)$$

Associated with this function, consider the following:

**Optimization problem:**

$$z^* := \max_{x \in X} \{H(x) : x \text{ satisfies the constraints}\} . \quad (35)$$

**Theorem 9.1**

*Decision  $x \in X$  is an optimal solution to the Optimization Problem iff it is a solution to the Satisficing Problem.*

**Proof.** This follows immediately, by inspection, from the definition of the *Satisficing Problem* and function  $H$ . Note that if we know a priori that there is a solution to the *Satisficing Problem*, then we can simplify the *Optimization Problem* (35) to  $\max_{x \in X} H(x)$ .

In short, the *Satisficing Problem* is a simple optimization problem whose objective function can take at most two values. One indicating that the constraints are satisfied, the other indicating that the constraints are violated. In practice, however, it is often possible to formulate more informative/useful objective functions for satisficing problems of this type.

The implication therefore is that when presented with say, a management problem, the question that we would face is not whether we should act as *satisficers* or as *optimizers*. Rather, the question we would have to grapple with is *what* elements of the problem should we seek to optimize and *what* elements should we seek to satisfy. In a similar vein, when it comes to the search for robustness, the question is not whether we should seek to robust-satisfice or to robust-optimize. Rather, the two questions that we would have to consider are these:

- What measure of robustness should be used with respect to the objective function(s), if any?
- What measure of robustness should be used with respect to the constraints, if any?

That said, consider now the rhetoric in Ben-Haim (2012) on the satisficing vs optimizing issue:

The investor who satisfices (rather than maximizes) can choose the alternative that would yield the required return over the greatest range of uncertain future scenarios. That is, the investor foregoes some aspiration for profit in exchange for some robustness against unacceptably low returns. In other words, satisficing is more robust to uncertainty than optimizing. Hence, this strategy is called robust-satisficing. If satisficing—rather than maximizing—is in some sense a better bet, then it will tend to persist under uncertain competition.

Ben-Haim (2012, p. 3)

This analysis is clearly reminiscent of the discussions on this and related issues that were in vogue in the 1960s which, since then, have been largely forgotten. To explain why this is so, one can do no better than to quote from the article *On the Techniques of Optimizing and Satisficing* (Odhnoff 1965). The abstract reads as follows:

In this paper I am going to make a comparison between the techniques of optimizing and satisficing with special reference to their use in business economics. Parts of the discussion on these techniques have been confused by the tendency to use the words ‘optimizing’ and ‘satisficing’ without reference to a particular model<sup>2</sup>. To avoid this confusion I have made the comparison on the level “choice of a model”.

As a basis for this comparison I have first chosen some thoughts about optimizing and satisficing given in business economics. Second I have in a formalised language described the situation of the decision maker when optimizing and satisficing, respectively, in a certain common base model. Here it is necessary to stress that it is my *interpretation* of Simon’s ideas on satisficing, that I give in this model<sup>3</sup>.

To elucidate this technique of satisficing in greater detail, I present, in section 1.3., a simple example connected to a well-known problem of choosing product-mix. Finally, in section 2 the comparison is made.

Odhnoff (1965, p. 39)

And in the last paragraph we read:

### 2.3. Concluding words

It seems meaningless to draw more general conclusions from this study than those presented in section 2.2. Hence, that section maybe the conclusion of this paper. In my opinion there is room for both ‘optimizing’ and ‘satisficing’ models in business economics. Unfortunately, the difference between ‘optimizing’ and ‘satisficing’ is often referred to as a difference in the quality of a certain choice. It is a triviality that an optimal result in an optimization can be an unsatisfactory result in a satisficing model. The best things would therefore be to avoid a general use of these two words.

Odhnoff (1965, p. 39)

But the lopsided discussion about the old and deservedly largely forgotten debate about satisficing vs optimizing, and the alleged superiority of satisficing over optimizing, continues to rage in Schwartz et al. (2011) and Ben-Haim (2012). This is yet another example of how misleading the info-gap rhetoric is! Why are there no references in these articles to critical assessments of Simon’s seminal work? Specifically, why are there no references to positions arguing that satisficing is not “superior” to optimizing?

As indicated by Odhnoff (1965), “general” rhetorics about the superiority of satisficing over optimizing fails to address the real issue. Indeed, a formal, rigorous treatment of the subject, gives a different picture altogether. And to illustrate, consider the following extracts from the article *Satisficing and Optimality* (Byron 1998):

A natural demand is that instrumentally rational actions implement the best means to one’s given ends. Optimizing conceptions of rationality endorse this demand. A competing conception of rationality—the satisficing conception—weakens this requirement and permits some rational actions to implement (merely) satisfactory means to the agent’s given ends. The present article argues that

instrumentalist theories of rationality as commonly understood cannot consistently accommodate this satisficing conception of rationality.

Byron (1998, p. 67)

So although at the local level rational satisficers might *appear* less planful, since they choose the first satisfactory option, the mere fact that they are satisficing at all indicates that satisficing is optimific and part of a (perhaps tacit) plan. If practical rationality is strictly instrumental, then rational agents are all nasty utility-ekers. Whatever force this objection has against optimizing accounts of rationality, it retains that force against satisficing theories.

Byron (1998, p. 93)

The inference is clear. Info-gap adherents who have been fed on a diet of “satisficing is superior to optimizing” would do well to obtain a second, better considered opinion on this matter. A good start would be Byron (2004).

Next consider this:

But academic economists seem to take scarce notice of Simon’s work<sup>(2,3)</sup>. Like Twain’s quip about the weather, they all talk about it (either weather or satisficing) but they do not do a damn thing. Rationality, we learn, is the optimization of profit or utility.

Ben-Haim (2012, p. 2)

I am not an economist, still this sounds like a sweeping statement. For, consider for instance this passage from WIKIPEDIA:

### **Economics**

In economics, satisficing is a behavior which attempts to achieve at least some minimum level of a particular variable, but which does not necessarily maximize its value. The most common application of the concept in economics is in the behavioral theory of the firm, which, unlike traditional accounts, postulates that producers treat profit not as a goal to be maximized, but as a constraint. Under these theories, a critical level of profit must be achieved by firms; thereafter, priority is attached to the attainment of other goals.

<http://en.wikipedia.org/wiki/Satisficing#Economics>

Read on June 18, 2012.

In any case, hasn’t it occurred to Ben-Haim (2012) that academic economists may not be doing a damn thing about “satisficing” (other than talk about it) simply because they do not find this approach to economic decision-making satisfactory?

For example, consider the abstract and concluding paragraph of the article *Maximising and satisficing* apparently written by an academic economist and published in *Journal of Economic Psychology*:

In this paper a proposition is defended that there is no real contradiction between choice theoretic ‘maximising’ notions and behaviourist ‘satisficing’ principles. If the often pronounced behaviourist critique of the maximising postulate is compared with well-designed choice theoretic models, then the apparent contrasts disappear. Behaviourist as well as choice theoretic frameworks permit the introduction of uncertainty and routines. In fact, both approaches lead to comparable results. Maximising and satisficing decision rules are equivalent rather than opposite principles.

van Witteloostuijn (1988, p. 289)

The right question to ask is *what* (individual or group) decision makers maximise or satisfice (see Rachlin 1980; Lea et al. 1987). The quest for and investigation of the ends that drive economic behaviour offers rich opportunities to economic psychology.

van Witteloostuijn (1988, p. 309)

Indeed, it seems that Schwartz et al. (2011) and Ben-Haim (2012) are unaware of the fact that there is a branch of optimization theory that is dedicated to **optimization under constraints**, which provides a framework for incorporating “satisficing” considerations, namely constraints, in optimization models.

### The case of the robust-satisficing squirrels

According to Ben-Haim (2012), squirrels apparently know better than some academic economists:

Biological evolution is a powerful metaphor for economics. Consider a squirrel nibbling acorns, and noticing a stand of fine oaks in the distance. There are probably better acorns there, but also other squirrels and predators. How long should the squirrel forage here before moving there? What strategy should guide the decision? The squirrel needs a critical amount of energy to survive the night. Maximizing caloric intake is *not* necessary. Maximizing the reliability of achieving the critical intake *is* necessary. What is maximized is not the substantive “good” (calories), but confidence in satisfying a critical requirement.

Ben-Haim (2012, p. 2)

This story gives a vivid illustration of Othnoff’s (1965) observation that “...It is a triviality that an optimal result in an optimization can be an unsatisfactory result in a satisficing model ...”. Thus, the fact that in certain situations a squirrel, call it *SQ*, finds it more appropriate to maximize the reliability of achieving a critical intake, than to maximize the caloric intake, does not mean that, under different conditions, *SQ* may not find it more appropriate to maximize the caloric intake for that evening. What hindrance is there to draw a scenario where *SQ* prefers to maximize the caloric intake?

That said, for the benefit of squirrels in general and robust-satisficing squirrels in particular, these comments are in order.

First, the fact of the matter is that Ben-Haim’s (2012) squirrels turn out to be *optimizers* after all! Because, what they actually do is to **maximize** the reliability of achieving critical intake. So, this story does not support the claim that “satisficing is better than optimizing”. To be *True Satisficers*, Ben-Haim’s (2012) squirrels should aim to satisfice a critical caloric intake **and** a desirable level of reliability of achieving this critical intake. But they do not! Which means that they are not *satisficers*.

Second, given the decisive role that the value of the **critical intake** plays in the robust-satisficing approach advocated by Ben-Haim (2012), it is surprising that we are not informed on how exactly would robust-satisficing squirrels under the oak trees go about computing this critical value. It is also rather odd that the existence of such a critical value is unknown to the caloric intake maximizers.

Third, on what grounds does Ben-Haim (2012) claim that “... Maximizing the reliability of achieving critical intake *is* necessary...”? Is the claim here that this is mandatory under the oak trees? If so, how is it that the *optimizers* are not aware of this dictate? Does Ben-Haim (2012) claim that flying in the face of this regulation the *optimizers* will maximize the caloric intake come hell or high water?

Fourth, who has the authority to decide how squirrels measure/define **performance**?

For those squirrels who define performance as the “reliability of achieving a critical intake”, rather than “caloric intake”, maximizing performance is the very same thing as that which Ben-Haim (2012) argues *is* necessary. In fact, most of the squirrels I know would define performance as the “reliability of achieving the critical intake”, should they know for sure, as Ben-Haim (2012) apparently does, that **it is necessary to maximize this reliability**.

More generally, to the best of my knowledge, squirrels, like humans, deploy a variety of strategies, depending on the problems they face and depending on their attitudes to uncertainty and risk. In fact, I should not be surprised at all if it transpired one day that some squirrels use multi-criteria *Pareto-Optimization* models to solve their acorns collection strategy, where caloric intake and the reliability of achieving a critical intake, are just two objectives, out of many.

And in any case, it is a well-known fact that many squirrels do not use the caloric intake itself as a measure of performance, but rather the “utility” of caloric intake, where the utility function begins to decrease with the caloric intake once the latter reaches a certain critical level. Squirrels who maximize such utility functions do not necessarily maximize their caloric intake.

And last but not least, there are rumors that many squirrels conduct their day to day business, including caloric intake, according to a strategy that aims to maximize the probability of survival (as individuals and as a species). There is no reason to believe that these squirrels maximize the reliability of achieving a critical intake.

The point is then that it is pointless to engage in generalities—as Schwartz et al. (2011) and Ben-Haim (2012) indeed do—about the superiority of robust-satisficing over performance maximizing, without making it clear how performance is measured, what is being satisfied, what measure of robustness is used, how the uncertainty is quantified, and so on.

And to sum it all up, the musings in Schwartz et al. (2011) and Ben-Haim (2012) on “satisficing vs optimizing” belong in the 1960s. They take no account whatsoever of the thinking on this issue post 1960s and they certainly do not present a balanced account on the state of the art in this area.

## 10 So what about the elephant in the room?

The ultimate question that the preceding discussion raises is then this:

What should/can be done, constructively, about the huge elephant<sup>a</sup> in the robust-satisficing room?

This, no doubt, is a delicate question, not only for proponents of the robust-satisficing approach, but also for journals, such as *Risk Analysis*, that continue to provide a platform for a grossly misleading rhetoric on robust-satisficing.



<sup>a</sup>See [http://en.wiktionary.org/wiki/elephant\\_in\\_the\\_room](http://en.wiktionary.org/wiki/elephant_in_the_room)

The first step to take to remedy this situation, and this would indeed be a big leap forward, is to acknowledge the presence of this huge elephant in the room. Once the elephant’s presence is acknowledged, info-gap scholars may perhaps be more inclined to talk/write about the challenges that this elephant poses to the rhetoric on robust-satisficing, and they may even learn a thing or two from this experience.

I submit that this is by far the better alternative to maintaining the status quo where readers of peer-reviewed journals, such as *Risk Analysis*, are systematically being misled to believe that following the rhetoric on “satisficing is better than optimizing” of the 1960s, will lead them forward in their quest for new ideas for addressing the challenges posed by decision-making under severe uncertainty.

For the benefit of advocates of info-gap’s robust-satisficing approach, let us recall the article *Robustness and Optimality as Criteria for Strategic Decisions* that was published 40 years ago. Its abstract and its last paragraph read as follows:

The use of “optimality” as an operational research criterion is insufficiently discriminating. Ample evidence exists that for many problems simple optimization (particularly profit maximization) does not represent the aims of management. In this paper we discuss the nature of the problem situations for which alternative decision criteria are more appropriate. In particular the structure of strategic planning problems is analysed. The provisional commitment involved in a plan (in contrast to the irrevocable commitment of a decision) leads to the development of a particular criterion, *robustness*—a measure of the flexibility which an initial decision of a plan maintains for achieving near-optimal states in conditions of uncertainty. The robustness concept is developed through the case study of a sequential factory location problem.

Rosenhead et al. (1972, p. 413)

Robustness and stability are two criteria which are appropriate in particular circumstances. Optimality is a criterion which will continue to have wide and useful application. Our argument is that criteria must be matched to circumstances; that more criteria are available than are often considered; and that new criteria can be developed when the need exists. If the criteria are related to the real requirements of the problem situation, their novelty need not be a bar to their understanding and acceptance by management.

Rosenhead et al. (1972, p. 430)

Schwartz et al. (2011) and Ben-Haim (2012) apparently prefer to ignore the progress achieved in the area of *Robust Optimization* since the publication of Rosenhead et al. (1972) and similar articles (e.g. Gupta and Rosenhead 1968) more than 40 years ago.

But the basic facts are these: in 2001 info-gap decision theory re-invented a simple model of local robustness, which is an instance of Wald’s maximin model (circa 1940) that is known universally as the *radius of stability* model (circa 1960). At present, proponents of info-gap decision theory are engaged in a far more ambitious “back-to-the-future” project: the re-invention of a *Robust Optimization* approach that seeks decisions that—in the language of *Robust Optimization*—are model robust.

What’s next then?

If it is indeed true that history repeats itself, then proponents of info-gap decision theory are well on their way to re-inventing *Robust Optimization* itself!

Peer-reviewed journals, such as *Risk Analysis*, would therefore do well to be more vigilant in their reviewing process so as to prevent the dissemination of unsubstantiated, misleading rhetoric such as the info-gap’s rhetoric on robust-satisficing.

It is ironic that this type of rhetoric about fundamental issues of *risk analysis* continues to be promulgated from the pages of a peer-reviewed journal that specializes in ... *risk analysis!*

Go figure!

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