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Risk Analysis 101: Fooled by local robustness ... again!*

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Preface

It is remarkable to what length some risk analysts would go to argue for the use of info-gap's robustness model (circa 2000) as a tool for decision-making under severe uncertainty. This is remarkable because this model is a re-invented version of a model of *local robustness* known universally as the *Radius of Stability* model (circa 1960), which in turn is a simple instance of Wald's famous *Maximin* model (circa 1940). And to top it all off, this model is ... utterly unsuitable for the management of a severe uncertainty of the kind that info-gap decision theory claims to deal with.

But it is even more remarkable that peer-reviewed journals, such as *Risk Analysis*, continue to prove defenseless against the misleading rhetoric and demagoguery that obscure these hard facts. So, in this discussion I examine the latest fallacy on info-gap's robustness model that has made its way into *Risk Analysis:*

> The analysis of a continuum of uncertainty from local to global is one of the novel ways in which info-gap analysis is informative.
>
> Hall et al. (2012)

As I show here, the secret weapon that Hall et al. (2012) attribute to info-gap's robustness model, which purportedly empowers it to perform this extraordinary feat, turns out to be the tried and tested *lamppost strategy:*

> Search for your lost keys under the nearest lamppost rather than in the dark alley where they were actually lost.

In the case of Hall et al. (2012), the application of the *lamppost strategy* to perform the alleged "analysis of a continuum of uncertainty from local to global" comes down to ... **a relaxation of the performance requirement to a degree that it is rendered irrelevant**. Thus "global" robustness is attained—surprise, surprise!—when the performance requirement is ... **redundant!** In short, Hall et al.'s (2012) argument boils down to this:

> In cases where the **robustness constraint is redundant,** info-gap's measure of local robustness becomes global.

A "novel" and "informative" analysis indeed!

In the article *Fooled by local robustness,* that was published recently in *Risk Analysis*, I advised readers of this journal of the fundamental flaws afflicting info-gap decision theory, calling special attention to the misleading rhetoric in the info-gap literature on the scope of

*This article was written for the **Risk Analysis 101 Project** to provide a Second Opinion on an article (Hall et al. 2012) that was published recently in the journal *Risk Analysis.* See Risk-Analysis-101.moshe-online.com.

info-gap's robustness model, its alleged "novelty", and so on. And yet, statements to this effect are repeated in Hall et al. (2012).

I therefore take the liberty—again—to call upon the Editorial Board of *Risk Analysis* to require authors of articles advocating the use of info-gap decision theory, to prove the validity of the claims that they make about this theory, by means of rigorous formal arguments. I also call upon the Editorial Board to require that articles on this theory properly represent the state of the art in the broad area of decision-making, robust optimization, robust control, and so on.

This call also applies to a new article *Doing Our Best: Optimization and the Management of Risk* (Ben-Haim 2012) that was also published recently in *Risk Analysis*, where the robust-satisficing approach proposed by info-gap decision theory is presented without the slightest reference to the field of *Robust Optimization* in spite of the fact that this approach is subsumed by *Robust Optimization* as a simple, indeed naive, *Robust Optimization* approach. An analysis of the misleading info-gap rhetoric on robust-satisficing can be found in *Robust Optimization: the elephant in the robust-satisficing room* (Sniedovich 2012c).

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1 Introduction

In the article *Fooled by local robustness* (Sniedovich 2012a), that was published recently in *Risk Analysis*, I explain in detail why info-gap's robustness model is a model of *local* robustness and why it is therefore utterly unsuitable for the treatment of the severe uncertainty that is postulated by info-gap decision theory (Ben-Haim 2001, 2006, 2010).

Since then, a new fallacy concerning the capabilities of info-gap's robustness analysis has surfaced in an article (Hall et al. 2012) that was published recently in *Risk Analysis*. In this discussion I give a terse explanation of this fallacy.

To be able to appreciate the full dimensions of this latest fallacy, it is important to have a clear idea of the nature of info-gap's robustness analysis, especially the **definition** of info-gap robustness. Keep in mind then that info-gap decision theory defines the robustness of decision $x \in X$ as follows (Ben-Haim 2001, 2006):

$$\hat{\alpha}(x, r_c) := \max_{\alpha \geq 0} \left\{ \alpha : r_c \leq \min_{u \in U(\alpha, \tilde{u})} r(x, u) \right\} \tag{1}$$

$$= \max_{\alpha \geq 0} \{ \alpha : r_c \leq r(x, u), \forall u \in U(\alpha, \tilde{u}) \} \tag{2}$$

where

- $\mathscr{U}$ = set of all possible/plausible values of parameter u under consideration.
- $\tilde{u}$ = *point estimate* of the true value of u.
- $U(\alpha, \tilde{u})$ = *neighborhood* of size (radius) α around $\tilde{u}$.
- $r(x, u)$ = *performance level* of decision x associated with the uncertainty parameter u.
- r_c = *critical level of performance.*

In words: the info-gap robustness of decision x, denoted $\hat{\alpha}(x, r_c)$, is the size (α) of the largest neighborhood $U(\alpha, \tilde{u})$ around $\tilde{u}$, all of whose elements satisfy the performance requirement $r_c \leq r(x, u)$ for a given *critical level of performance* r_c.

No amount of rhetoric can change this fact.

Clearly, you don't have to be a risk analyst to immediately see that a robustness analysis operating according to this definition yields *local robustness,* namely a measure of robustness of this type is inherently *local.* Should you have any doubts about this fact, take a look at Figure 1.

This figure displays the info-gap robustness of two decisions for the same value of $\tilde{u}$, the same value of r_c, and the same uncertainty space $\mathscr{U}$. For each decision, the shaded area represents the set of values of u that satisfy the performance constraint $r_c \leq r(x, u)$, and the circles represent neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$ around $\tilde{u}$. The info-gap robustness of the two decisions is represented by the radii of the thick (blue) circles that are tangent to the boundary between the shaded areas and the white areas of the respective decisions.

As this figure clearly brings out, decision x'' is deemed much more info-gap robust than decision x' at $\tilde{u}$, even though the shaded area associated with decision x' is much larger than the shaded area associated with decision x''. In other words, although the set of acceptable values of u associated with decision x' is much larger than the set of acceptable values of u associated with decision x'', info-gap's robustness analysis decrees that decision x'' is more robust than decision x' for the given values of r_c and $\tilde{u}$.

This determination of robustness clearly and unambiguously attests to the model's mode of operation. The analysis prescribed by this model measures the *local* robustness of decision x against **small deviations/perturbations** in the value of $\tilde{u}$. It does **not** measure the global robustness of decision x against variations in the value of u over $\mathscr{U}$. In a word, by definition info-gap robustness is a measure of *local* robustness.

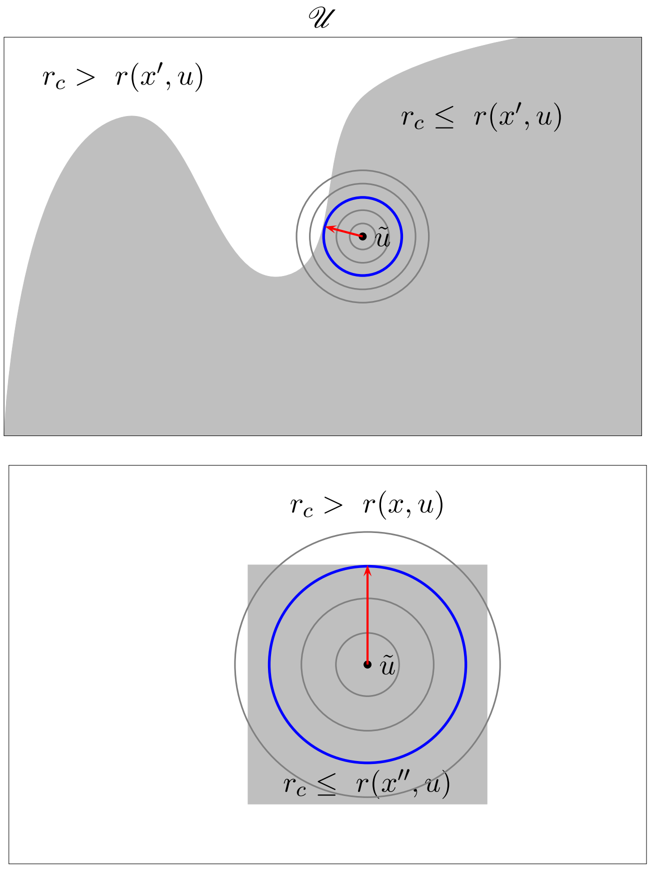

Figure 1: Info-gap robustness of two decisions $\tilde{u}$ for a given value of r_c

2 A stroll towards ∞ with info-gap robustness

And yet, despite it being clear as daylight that info-gap's robustness analysis is manifestly inherently *local,* Hall et al. (2012) make this astounding claim:

> For small α, searching set $U(\alpha, \tilde{u})$ resembles a local robustness analysis. However, α is allowed to increase so that in the limit the set $U(\alpha, \tilde{u})$ covers the entire parameter space and the analysis becomes one of global robustness. The analysis of a continuum of uncertainty from local to global is one of the novel ways in which info-gap analysis is informative.
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> Hall, J., Lempert, R.J., Keller, K., Hackbarth, C., Mijere, A., and McInerney, D.J. (2012, p. 6)
> Robust climate policies under uncertainty: A comparison of Info-Gap and RDM methods
> *Risk Analysis* (Early View)

The question is therefore this: how could Hall et al. (2012) possibly arrive at this conclusion when it is so obviously in error?

The explanation is this.

Hall et al. (2012) are clearly unaware, or perhaps forgot, or prefer to gloss over, or ...whatever, that pursuant to info-gap's definition of robustness (1), in order to increase the range of admissible values of α, the value of the critical performance level, r_c, **must be decreased** accordingly, so as to comply with the robustness constraint

$$r_c \leq \min_{u \in U(\alpha, \tilde{u})} r(x, u) \ . \tag{3}$$

Thus, for a given value of r_c, the admissible values of α are bounded above by the info-gap robustness of decision x, namely by $\hat{\alpha}(x, r_c)$. This fact is broadcast loud and clear by info-gap's definition of robustness (1), so much so that one need not be a risk analyst to see that the range of admissible values of α associated with decision x is $[0, \hat{\alpha}(x, r_c)]$.

Indeed, if, as claimed by Hall et al. (2012), α is not bounded above, how is it then that the definition of info-gap robustness (1) prescribes the **maximization** of this very α? After all, the whole point of info-gap's robustness analysis of decision x is to find the value of this upper-bound, because this value is precisely the info-gap robustness of decision x for the given values of r_c and $\tilde{u}$!

This fact is made clear by the Father of info-gap decision theory himself (emphasis added):

> This approach is quite different from a max-min strategy wherein one maximizes a minimum profit. What one maximizes here is the robustness to uncertainty, making this strategy attractive for avoidance of uncertainty. **Of course, one can still gamble since setting a high value for the least acceptable profit r_c will reduce the available robustness $\hat{\alpha}$.**
>
> Ben-Haim (2006, p. 93)

In short:

> In the context of info-gap's robustness analysis of decision x, the range of admissible values of α for a given value of r_c is $[0, \hat{\alpha}(x, r_c)]$. Hence, to increase this range, namely to increase the value of $\hat{\alpha}(x, r_c)$, the value of r_c must be decreased.

To set the stage for my explanation of this blunder, I must first point out the following fact. Hall et al. (2012) appear to confound two types of robustness that can be associated with decision x:

- The info-gap robustness of decision x, as stipulated by (1), which is the local robustness of x with respect to the **performance constraint** $r_c \leq r(x, u)$ for a given value of r_c and a given value of $\tilde{u}$.
- The (local) classic security level robustness of decision x, which is the worst-case robustness of x with respect to the **reward** $r(x, u)$, defined by the right hand side of (3), namely

$$SL(x, \alpha) := \min_{u \in U(\alpha, \tilde{u})} r(x, u) \ , \ x \in X, \alpha \geq 0 \ . \tag{4}$$

  The scope of this robustness analysis is determined by the "size" of $U(\alpha, \tilde{u})$, namely by the value of α, and it is independent of the value of r_c.

The difference between these two types of robustness is clearly manifested in the resulting robustness *curves* of decision x.

- The info-gap robustness curve of decision x shows the variation of the info-gap robustness of decision x, namely $\hat{\alpha}(x, r_c)$, in relation to the variation of the value of r_c. And, the important point to note here is that each value of $\hat{\alpha}(x, r_c)$ on this curve is the **local** robustness of decision x, as determined by the corresponding value of r_c, according to (1). **It is not the global** robustness of x with respect to $r_c \leq r(x, u)$ over the parameter space $\mathscr{U}$.
- The (local) classic security level robustness curve of decision x shows the variation of the (local) classic security level robustness of decision x, namely $SL(x, \alpha)$, in relation to the variation in the value of α. In this case as well, each point on this curve is the local security level robustness of decision x, as determined by (4), for the corresponding value of α.

Hall et al.'s (2012) confusion of the two types of robustness seems to be due to the fact that the robustness curves generated by these two robustness analyses are the same, more accurately these curves are *pseudo-inverse* of each other. So, to Hall et al. (2012) *info-gap robustness* and *classic security level robustness* seem to be one and the same thing.

And this takes us straight to what seems to be the crux of the above quoted claim, namely Hall et al.'s (2012) contention that "in the limit", presumably where $\alpha = \infty$, the info-gap robustness of decision x is global. Let us then examine what happens on the road to this limit, and at the limit itself.

2.1 On the road to the promised limit

There are three fundamental errors in the proposition that "in the limit", the info-gap robustness of decision x is global rather than local. The first is quite obvious: there are cases where the limit simply cannot be reached. The second is even more obvious: by changing the value of r_c we effectively change the robustness problem under consideration, so that the best that we can hope for is to obtain the correct answer to the wrong robustness question. The third is a bit more subtle and it has to do with the type of result that is actually yielded in the limit, in cases where it can be reached.

But, what Hall et al.'s (2012) contention really betrays is a deeper methodological misconception about the role of r_c, and the structure of the performance constraint $r_c \leq r(x, u)$, in info-gap decision theory.

2.1.1 Fixed value of r_c

As was indicated above, it is impossible to change the scope of info-gap's robustness analysis (increase) the range of admissible values of α, without changing (decreasing) the values of r_c. But, the important point to note in this regard is that there are cases where the value of r_c cannot be changed because it is fixed in advance to the effect that it is outside the decision maker's control:

> Performance requirements may originate in various ways: by legislation, by administrative fiat, by public debate and collective decision making, and so on. Furthermore, multiple constraints of various sorts may be imposed, such as cost and safety constraints, engendering trade-offs.
>
> Ben-Haim (2012, p. 3)
> Doing Our Best: Optimization and the Management of Risk.
> *Risk Analysis* (Early View)

So, the question is this: what do Hall et al. (2012) propose to do in cases where the value of r_c is fixed in advance, say to $r_c = 37$, so that it cannot be changed, to thereby prevent the analysis from proceeding to the "promised limit"? Similarly, what do Hall et al. (2012) propose to do in cases where the range of admissible values of r_c is small, to equally prevent the analysis from reaching the "promised limit"? And what do Hall et al. (2012) propose to do in cases where there are two performance functions, say $\underline{r}$ and $\overline{r}$, and two performance constraints of the form $\underline{r}(x, u) \leq r_c \leq \overline{r}(x, u)$?

It is important to note that these questions, which show Hall et al's (2012) claim for what it is, arise not as a result of some "technical hitch" that can be given a "quick fix". They arise as a result of Hall et al.'s (2012) fundamentally, namely methodologically, erroneous portrayal of info-gap's robustness model: its mode of operation, its capabilities, hence its scope.

2.1.2 Under the Lamppost

As I pointed out above, by changing the value of r_c we effectively modify the robustness problem under consideration to obtain a new problem that we did not set out to solve. Thus, if we want to determine the info-gap robustness of decision x with respect to the performance constraint $248 \leq r(x, u)$, namely for $r_c = 248$, surely, it won't do us much good to establish the info-gap robustness of decision x with respect to the performance constraint $123 \leq r(x, u)$, namely for $r_c = 123$.

And in the same vein, if we seek to determine the global robustness of decision x with respect to the performance constraint $248 \leq r(x, u)$, namely for $r_c = 248$, it won't do us much good to establish the global robustness of decision x with respect to the performance constraint $123 \leq r(x,u)$, namely for $r_c = 123$.

But this is precisely what info-gap's global robustness analysis, as depicted by Hall et al. (2012), amounts to: $\boxed{\text{a lamppost strategy}}$ par excellence.

So my question to Hall et al. (2012) is this: if we want to determine the global robustness of decision x with respect to the requirement $248 \leq r(x,u)$, why should we settle for the global robustness of decision x with respect to the requirement $123 \leq r(x,u)$?

2.1.3 At the promised limit

What Hall et al. (2012) apparently do not realize is that once the promised "limit" ($\alpha = \infty$) is reached, what we find there is a value of r_c **so small** that the performance constraint $r_c \leq r(x,u)$ is in fact rendered **redundant.** That is, for this small value of r_c, decision x satisfies the performance constraint $r_c \leq r(x,u)$ for **all** $u \in \mathscr{U}$, which means that determining the robustness of decision x is not an issue to begin with, because total robustness over $\mathscr{U}$ is trivially obvious.

So, what Hall et al. (2012) tell us is this:

> In cases where the performance requirement $r_c \leq r(x,u)$ is $\boxed{\text{redundant,}}$ so that the robustness issue is a $\boxed{\text{non-issue,}}$ the inherently local info-gap robustness is global. For a sufficiently small value of r_c we have $r_c \leq r(x,u), \forall u \in U(\infty,\tilde{u}) = \mathscr{U}$.
>
> A "novel" and "informative" approach to robustness indeed!

To illustrate this point, consider the case where for some decision $x' \in X$, the range of values of $r(x',u)$ over $u \in \mathscr{U}$ is say the interval $[2, 1000]$, and the robustness question is as follows:

> What is the global robustness of decision x' with respect to the performance constraint $\boxed{348} \leq r(x',u)$ against variations in the value of u over $\mathscr{U}$?

Clearly, as an inherently local analysis, info-gap's robustness analysis does not address questions such as this, so that it cannot answer them. Instead, for a stipulated value of $\tilde{u}$, it can generate the info-gap robustness curve of decision x' which shows the variation of the local robustness of decision x' with respect to the varying value of r_c for the given value of $\tilde{u}$.

Observe next that as the range of values of $r(x',u)$ over $u \in \mathscr{U}$ is $[2, 1000]$, it follows that the info-gap robustness of decision x' is *unbounded* for all values of r_c such that

$$r_c \leq \boxed{2} = \min_{u \in \mathscr{U}} r(x',u) \ . \tag{5}$$

So, the only answer that info-gap's robustness analysis can possibly yield to the above simple question about the global robustness of decision x' over $\mathscr{U}$ is this:

> Decision x' is globally super-robust over $\mathscr{U}$ with respect to the performance constraint $\boxed{2} \leq r(x',u)$. That is, decision x' satisfies this constraint for all $u \in \mathscr{U}$.

In short, I ask the simple question: "what is the global robustness of decision x' with respect to $r_c = \boxed{348}$?" and the answer I get from info-gap decision theory spells out the global robustness of x' with respect to $r_c = \boxed{2}$. Namely, the answer I get is for a value of r_c which renders the constraint under consideration **redundant**, so that the solution can be determined by (5) without any recourse to info-gap decision theory **at all!**[1]

To repeat, an application *par excellence* of the proverbial lamppost search method to the task of identifying globally robust decisions!

[1]We do not need the point estimate $\tilde{u}$, we do not need the horizon of uncertainty α, and we do not need the neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$.

And to sum it all up: the great "novelty" that Hall et al. (2012, p. 6) identify in info-gap decision theory's robustness analysis turns out to be an ability to determine correctly the "global" robustness of a decision with respect to a performance constraint in situations where both the global and the local robustness of a decision are a non-issue because the performance constraint is **redundant.** Never mind that the very attribution of "novelty" to info-gap's robustness analysis is utterly misleading, considering that its robustness model is a simple **radius of stability** model (circa 1960)!

That said, it is instructive to identify the misconceptions that are at the bottom of Hall et al's. (2012) erroneous depiction of info-gap's robustness analysis.

3 "Robustness" in inverted commas

It seems that Hall et al. (2012) misconstrue the info-gap robustness curve, which shows the variation of the info-gap robustness of decision x, namely $\hat{\alpha}(x, r_c)$, relative to varying the value of r_c, for a capability of info-gap decision theory to control the scope of its robustness analysis of the performance constraint $r_c \leq r(x, u)$.

Hall et al. (2012) need reminding, therefore, that the points on this robustness curve **do not represent a continuum** from a local to a global robustness analysis of the performance constraint $r_c \leq r(x, u)$ for a given value of r_c. Rather, insofar as info-gap decision theory is concerned, the points on this curve are the result of a parametric analysis of the info-gap robustness of $\hat{\alpha}(x, r_c)$, which is governed by the critical level of the reward r_c. Thus, from this perspective, each value of $\hat{\alpha}(x, r_c)$ on this curve represents the *local* robustness of decision x relative to the performance constraint $r_c \leq r(x, u)$ for a given value of r_c.

It is important to make it clear therefore that:

Observation:
The global robustness that Hall et al. (2012, p. 6) allude to through the term "Robustness" (note the inverted commas) namely:

$$SL(x) := \min_{u \in \mathscr{U}} r(x, u) \ , \ x \in X \tag{6}$$

is not the info-gap robustness of decision x.

This measure of global "Robustness" (in inverted commas) is known universally as the security level of decision x, and it represents global robustness with respect to the reward $r(x, u)$, rather than global robustness with respect to the performance constraint $r_c \leq r(x, u)$.

Differently put, Hall et al.'s (2012) "Robustness" (in inverted commas), is not info-gap robustness, which is a performance constraint driven robustness. Hall et al.'s (2012) "Robustness" is the classic measure of robustness associated with classic *maximin models of the reward,* namely models seeking decisions with the highest security level:

$$z^* := \max_{x \in X} SL(x) \tag{7}$$

$$= \max_{x \in X} \min_{u \in \mathscr{U}} r(x, u) \ . \tag{8}$$

Thus, the info-gap robustness curve of decision x is in fact the pseudo-inverse of the classic decision theory robustness curve, which describes the variation of the (local) classic *security level* of decision x, defined in (4), in relation to varying the scope of the local robustness analysis (α). The associated *local* maximin model is as follows:

$$z^*(\alpha) := \max_{x \in X} SL(x, \alpha) \ , \ \alpha \geq 0 \tag{9}$$

$$= \max_{x \in X} \min_{u \in U(\alpha, \tilde{u})} r(x, u) \ . \tag{10}$$

By definition, $SL(x, \alpha)$ denotes the (local) classic $\boxed{\text{security level}}$ of decision x over the uncertainty space $V = U(\alpha, \tilde{u})$, for a given value of α. Surely, Hall et al. (2012) must know that $SL(x, \alpha)$ is not the info-gap robustness of decision x!!!

The interesting fact in this entire business is that for a fixed x, the curve of $SL(x, \alpha)$ as a function of α, is the *pseudo-inverse* of the curve of $\hat{\alpha}(x, r_c)$ as a function of r_c. This means of course that the so-called info-gap robustness curve can be constructed with no recourse whatsoever to the performance constraint $r_c \leq r(x, u)$, hence with no recourse whatsoever to "robust-satisficing". Or to put it more bluntly, there is no call whatsoever to use info-gap decision theory to construct the security level robustness curve for decision x.

It is therefore extremely odd that Hall et al. (2012) do not mention this inverse relation between info-gap robustness and the classic $\boxed{\text{security level}}$ robustness. And it is even more odd that instead of using the standard, conventional, well known term $\boxed{\text{security level}}$, pertaining to the famous, classic $\boxed{\text{maximin model,}}$ Hall et al. (2012) use the term "Robustness" (in inverted commas).

But, to return to what is of more immediate concern to us in this discussion. Hall et al. (2012) apparently misconstrue the capability of the classic maximin model of reward (8) to control the scope of its robustness analysis from local to global by varying the size of the uncertainty space $\mathscr{U}$ parametrically, as prescribed by (4), for a capability of info-gap's robustness model. They thus, **wrongly** attribute info-gap's robustness model the ability to control the scope of its robustness analysis of decision x with respect to the performance constraint $r_c \leq r(x, u)$ for a given value of r_c.

Incidentally, this misattribution is brought to an absurd level in the following statement:

> Info-gap generalizes the maximin strategy by identifying worst-case outcomes at increasing levels (horizons) of uncertainty. This permits the construction of 'robustness curves' that describe the decay in guaranteed minimum performance (or worst-case outcome) as uncertainty increases.
>
> Wintle et al. (2011, p. 357)

That is, not only that Wintle et al. (2011) misconstrue the capability of the classic maximin model of the reward (8) to control the scope of its robustness analysis by varying the size of the uncertainty space $\mathscr{U}$ parametrically, as prescribed by (4), for a capability of info-gap's robustness model. They go so far as to make the preposterous claim that Info-gap thereby " generalizes the maximin strategy".

It goes without saying that the maximin strategy does not require Wintle et al.'s (2011) "permission" to vary the size of the uncertainty space $\mathscr{U}$ parametrically, as prescribed by (4), to generate a curve showing the variation in a decision's security level relative to the variation in the size of the neighborhood over which the worst-case analysis is conducted.

If anything, it is Wintle et al.'s (2011) as well as Hall et al. (2012) who require reminding that info-gap's robustness curve is in fact the pseudo-inverse of the robustness curve associated with the classic security level of decisions.

Having said all that, I should point out, especially to those who are not familiar with the ins and outs of the info-gap story, that I have shown time and again that info-gap's robustness model (1) is a simple instance of Wald's generic maximin model (e.g. Sniedovich 2007, 2010, 2012, 2012a, 2012b), say this model:

$$z^*(x, \tilde{u}) := \max_{\alpha \geq 0} \min_{u \in U(\alpha, \tilde{u})} \{g(\alpha, u) : con(x; u), \forall u \in U(\alpha, \tilde{u})\} \ , \ x \in X \tag{11}$$

where g is a real-valued function on $[0, \infty) \times \mathscr{U}$ and $con(x; u)$ denotes a list of constraints on the (x, u) pairs[2].

All the same, info-gap's robustness model (1) does not have the capability to continuously change the scope of the robustness analysis from local to global as mistakenly claimed by Hall

[2]**Proof.** For the simple instance where $g(\alpha, u) \equiv \alpha$ and the list $con(x; u)$ consists of the single constraint $r_c \leq r(x, u)$, this maximin model simplifies to (1). (QED)

et al. (2012). This is a capability of the (local) *security level model* of the reward $r(x,u)$ over the neighborhoods $U(\alpha,\tilde{u}), \alpha \geq 0$, namely of the model specified by (4).

4 The role of r_c and $r_c \leq r(x,u)$ in info-gap decision theory

As indicate above, Hall et al.'s (2012) misguided claim, which attributes global capabilities to info-gap's robustness analysis, clearly hangs on the performance requirement being of the form $r_c \leq r(x,u)$ and, on the value of r_c not being fixed, thereby allowing us to decrease it at will.

But, it is important to note that although it is no doubt true that the format $r_c \leq r(x,u)$ dominates in info-gap publications, info-gap decision theory *per se* does not require that the performance requirements be of this form. In fact, the opposite is true, for recall what the Father of info-gap decision theory has to says on this matter:

> Performance requirements may originate in various ways: by legislation, by administrative fiat, by public debate and collective decision making, and so on. Furthermore, multiple constraints of various sorts may be imposed, such as cost and safety constraints, engendering trade-offs.
>
> Ben-Haim (2012, p. 3)
> Doing Our Best: Optimization and the Management of Risk.
> *Risk Analysis* (Early View)

Indeed, in Ben-Haim (2001, p. 34; 2006, p. 38), the robustness of decision q is expressed as follows:

$$\hat{\alpha}(q) = \max \{\alpha : \text{ minimal requirements are always satisfied}\}$$

More formally, let $req(q,u)$ denote the list of performance requirements associated with info-gap's robustness analysis, expressed in terms of the decision variable q and the uncertainty parameter u. Then, according to info-gap decision theory, the robustness of decision q would be defined as follows:

$$\hat{\alpha}(q) := \max_{\alpha \geq 0} \{\alpha : req(q,u), \forall u \in U(\alpha,\tilde{u})\} . \tag{12}$$

Observe that in this framework the admissible values of α are bounded above by $\hat{\alpha}(q)$, so that the range of admissible values of α is $[0, \hat{\alpha}(q)]$. Take note that this interval is not expressed explicitly in terms of a parameter of the performance requirements $req(q,u)$. This clearly disallows increasing this range parametrically in a manner similar to that of the simple case where the performance requirement is $r_c \leq r(q,u)$, hence the simple case where the range of admissible values of α is $[0, \hat{\alpha}(q, r_c)]$, and $\hat{\alpha}(q, r_c)$ is non-increasing with r_c.

And all this goes to show that the claim:

> For small α, searching set $U(\alpha,\tilde{u})$ resembles a local robustness analysis. However, α is allowed to increase so that in the limit the set $U(\alpha,\tilde{u})$ covers the entire parameter space and the analysis becomes one of global robustness. The analysis of a continuum of uncertainty from local to global is one of the novel ways in which info-gap analysis is informative.
>
> Hall et al. (2012, p. 6)

is not only grossly in error technically, it betrays a deeper methodological misapprehension of info-gap's robustness analysis, and of info-gap decision theory as a whole. The same applies to the claim by Wintle et al. (2011) that info-gap decision theory generalizes the maximin strategy.

5 Severe uncertainty according to info-gap decision theory

Finally, it is important to take another look at the inherently local nature of info-gap's robustness analysis so as to demonstrate how utterly unsuitable this model is for the management of a severe uncertainty of the type that info-gap decision theory claims to address.

A big fuss is made in the info-gap literature about info-gap's robustness model providing a reliable tool for the modeling, analysis and management of decision problems where the severity of the uncertainty is characterized by:

- A vast (e.g. unbounded) uncertainty space.
- A poor point estimate that can be substantially wrong. It is a sort of "guess", sometimes a "poor guess", even a "wild guess".
- A likelihood-free quantification of uncertainty.

Indeed, according to the Father of info-gap decision theory:

> Most of the commonly encountered info-gap models are unbounded.
>
> Ben-Haim (2006, p. 210)

But, Figure 2 gives a vivid illustration of the total incongruity between this rhetoric and the local nature of info-gap's robustness model, namely its manifest inability to handle a severe uncertainty of the type it postulates.

In this figure, the large rectangle represents a small part of the **unbounded** uncertainty space $\mathscr{U}$ pertaining to the most commonly encountered info-gap models, and the small white circle represents the neighborhood $U(\hat{\alpha}(x,r_c),\tilde{u})$ for decision $x \in X$.

It is important to realize that the troubles besetting info-gap's robustness model are due not only to the fact that under severe uncertainty (e.g. unbounded uncertainty space) the neighborhood $U(\hat{\alpha}(x,r_c),\tilde{u})$ can be markedly smaller than the uncertainty space $\mathscr{U}$. An important issue here is that because the "sample" represented by the elements of the set $U(\hat{\alpha}(x,r_c),\tilde{u})$ consists of elements taken only from one "locale" in $\mathscr{U}$, this sample does not adequately represent the scope of variations in the value of u over $\mathscr{U}$.

Indeed, the info-gap robustness of decision x, namely the value of $\hat{\alpha}(x,r_c)$, **takes no account whatsoever** of the performance of decision x over a vast (unbounded) subset of the uncertainty space $\mathscr{U}$. In other words, the value of $\hat{\alpha}(x,r_c)$ **completely ignores** the performance of decision x over the vast set $NML(q) := \mathscr{U} \setminus U(\alpha^*,\tilde{u})$, where $\alpha^* = \hat{\alpha}(x,r_c) + \varepsilon$ and $\varepsilon > 0$ can be arbitrarily small. This vast set, which I call *No Man's Land* (see Snieovich 2010, 2012, 2912a, 2012b), is represented by the black area in the figure.

If this is not *voodoo* decision-making, what is?!

6 Conclusions

- Info-gap's robustness model is a model of *local robustness par excellence.*
- Hall et al.'s (2012) pronouncements on the capabilities of info-gap's robustness analysis rest on a profound misapprehension of this analysis, notably the role that α plays in this analysis.
- Hall et al. (2012) misconstrue the parametric analysis of $\hat{\alpha}(x,r_c)$ with respect to r_c from large to small values of r_c, as an ability of info-gap's robustness analysis to control the scope of its robustness analysis so as to allow varying it at will from local to global.
- The "Robustness" (note the inverted commas) considered by Hall et al. (2012, p. 6) is not info-gap's robustness. It is a measure of robustness that is known universally as *security level* and is associated with maximin models that seek worst-case robustness with respect to the *reward* $r(x,u)$, not the constraint $r_c \leq r(x,u)$.
- Info-gap's robustness model does not have the capability to change the scope of the robustness analysis from local to global without changing the robustness problem under consideration, by relaxing the performance requirement $r_c \leq r(x,u)$ to such an extent that it becomes *redundant.*

Figure 2: Robustness analysis of most of the commonly encountered info-gap models

- The capability to continuously change the scope of the robustness analysis from local to global is mistakenly attributed by Hall et al. (2012) to info-gap's robustness model. This is a capability of the (local) *security level model* of the reward $r(x, u)$ over the neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$.
- The "limit" case, that presumably corroborates Hall et al.'s (2012) contention, is vacuous, if not inane. It represents situations where the performance constraint $r_c \leq r(x, u)$ is *redundant,* namely situations where robustness is not an issue. If anything, by resorting to this inane case Hall et al. (2012) in fact amplify the profoundly inherently local nature of info-gap's robustness analysis.
- It is high time that info-gap advocates faced up to the fact that info-gap robustness model is not novel, that it is a re-invented version of the *radius of stability* model (circa 1960), hence a simple instance of Wald's famous maximin model (circa 1940).

7 Postscript

Info-gap proponents should take note that a rhetoric that misrepresents the basic facts about info-gap's robustness model is futile and counter-productive.

Peer-reviewed journals, such as *Risk Analysis,* would do well to be more vigilant in their reviewing process so as to prevent the dissemination of an unsubstantiated, misleading rhetoric such as the info-gaps rhetoric on robust-satisficing and decision-making under severe uncertainty.

For the benefit of readers of publications on info-gap decision theory, I posted on my website[3] more than 30 reviews of publications, mostly peer-reviewed articles, on this theory.

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[3]See http://info-gap.moshe-online.com/reviews.html