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Risk Analysis 101 Series

The latest info-gap/risk-analysis rhetorics

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Preface

It is hard to explain the steady stream of articles on info-gap decision theory in *Risk Analysis: An International Journal* (henceforth RISA), over the past five years, without assuming that the referees and editors of this journal must have proven utterly defenseless before the rhetoric that is the stock in trade of this theory. It is particularly hard to comprehend how the discussions on this theory, in two articles that were published in a recent issue of RISA, namely Volume 32, Issue 10 (henceforth ISSUE), managed to pass muster in the peer-review process of this journal.

But pass they did!

The reason that I am putting this matter this bluntly is to point out that in the article *A bird's view of info-gap decision theory* (Sniedovich 2010), published in the journal *Risk Finance,* I explained in considerable detail the egregious flaws in info-gap decision theory, calling particular attention to the misleading rhetoric on this theory in the risk analysis literature.

As the flow of articles on this theory continued unabated in RISA, I submitted an article entitled: *Fooled by local robustness* to RISA, which, after some toing and froing, was accepted for publication in RISA (Sniedovich 2012a). In this article I outlined, again, some of the flaws afflicting info-gap decision theory. The abstract of this article reads as follows:

> One would have expected the considerable public debate created by Nassim Taleb's two best selling books on uncertainty, *Fooled by Randomness and The Black Swan,* to inspire greater caution to the fundamental difficulties posed by *severe uncertainty.* Yet, methodologies exhibiting an incautious approach to uncertainty have been proposed recently in a range of publications. So, the objective of this short note is to call attention to a prime example of an incautious approach to severe uncertainty that is manifested in the proposition to use the concept *radius of stability* as a measure of robustness

> against severe uncertainty. The central proposition of this approach, which is exemplified in *info-gap decision theory,* is this: use a simple radius of stability model to analyze and manage a severe uncertainty that is characterized by a vast uncertainty space, a poor point estimate, and a likelihood-free quantification of uncertainty. This short discussion serves then as a reminder that the generic radius of stability model is a model of *local* robustness. It is, therefore, utterly unsuitable for the treatment of severe uncertainty when the latter is characterized by a poor estimate of the parameter of interest, a vast uncertainty space, and a likelihood-free quantification of uncertainty.
>
> Sniedovich (2012a, p. 1630)

The point is though that this article was published alongside two other articles that not only advocate the use of info-gap decision theory, their advocacy of this theory is couched in terms of the same grossly misleading rhetoric that my article had exposed for what it is. What is more, the editorial introducing the ISSUE appears to endorse info-gap decision theory, and what is even more unfortunate, it obfuscates on my "warnings" against going down "the wrong path" (emphasis added):

> **From the Editors**
>
> Deep uncertainty is one of the most undefeatable challenges of risk analysis. This issue features four deep uncertainty contributions. Tony Cox provides a introduction that summarizes the issue and then assesses the methods and how they may be most effectively used. We recommend his paper as a good reference point for those interested in deep uncertainty. **Moshe Sniedovich's "Perspective" article warns us that some deep uncertainty methods can take us down the wrong path.** Yakov Ben-Haim has designed and tested methods to account for deep uncertainty. His commentary discusses concepts, ethics, and optimization tools. He argues for satisficing solutions, rather than optimizing ones, and he offers examples to support his arguments.
>
> Jim Hall et al. use the issue of global climate change as an illustration of deep uncertainty. Using the info-gap method developed by Ben-Haim and the robust decision-making (RDM) method developed by Robert Lempert, Steven Popper, and Steven Bankes, they carefully discuss the similarities and differences between the methods. When applied to climate change, the two methods yield relatively similar policy recommendations. However, they observe interesting differences, and conclude that the use of both methods helps to elucidate policy options. As our global challenges intensify, we will increasingly confront deep uncertainty.
>
> Michael Greenberg and Karen Lowrie (2012, p. 1605)

I am referring of course to the statement:

> **Moshe Sniedovich's "Perspective" article warns us that some deep uncertainty methods can take us down the wrong path.**

Surely, these warnings were not directed at some anonymous "methods", but specifically and primarily at . . . **info-gap decision theory,** the very theory that is advocated in Ben-Haim (2012a) and Hall et al. (2012) in the same ISSUE of the journal!

In other words, Sniedovich's article clearly and unambiguously warns that it is **info-gap decision theory that takes us down the wrong path**!

It would have therefore been far more informative, indeed more accurate, to state that:

> **Moshe Sniedovich's "Perspective" article warns us that some deep uncertainty methods, such as info-gap decision theory, can take us down the wrong path.**

Furthermore, some indication could have been given to the fact that Sniedovich's perspective article warns that info-gap robustness is no more and no less than a **reinvented wheel**. Namely, that this article proves, formally and rigorously, that info-gap robustness is a re-invention of a staple concept known universally as **radius of stability** (circa 1960), which for decades, has been used extensively in many fields, as a measure of **local** stability/robustness against **small perturbations** in a nominal value of a parameter.

Indeed, some suggestion could have been made that Sniedovich's perspective warns that the trouble with info-gap decision theory is that its local robustness analysis is utterly unsuitable for the treatment of the severe uncertainty **that it claims to address.**

But, there is nothing in the editorial quoted above to so much as hint at these facts. To the contrary, this editorial seems to continue in the same vein of providing a platform for unsubstantiated, erroneous, misleading rhetoric on the management of severe uncertainty in general and on info-gap decision theory in particular.

One could have argued therefore that, given this state of affairs, there seems to be little point in raising this matter, **again,** with RISA referees and editors, as arguably everything that should have been said on this matter has already been said.

Still …

There are **new claims** in the ISSUE that call, indeed cry out, for a detailed commentary, because claims such as these clearly have no place in a refereed journal. Hence this discussion. A more comprehensive discussion on this topic can be found in the article *Fooled by info-gap decision theory* (Sniedovich 2013b).

In this discussion I illustrate some of the inevitable consequences of the failure of RISA's peer-review process to detect not only the profound methodological and conceptual flaws in the theory, but also obvious egregious **technical errors** that should have been easily identified by the referees for what they are, namely: obvious, serious, technical errors.

More specifically, I focus here on those manifest technical errors that form part of the misleading rhetoric in the info-gap literature, including the ISSUE, whose objective is to "substantiate" the claims that:

- Info-gap's robustness analysis can handle an unbounded uncertainty, whereas a maximin analysis requires the uncertainty to be bounded (Ben-Haim 2012a).
- Info-gap robustness is not a measure of local robustness (e.g. Ben-Haim 2012a, Hall et al. 2012). What is more, info-gap decision theory can change the scope of its robustness analysis from local to global (Hall et al. 2012).

Given the magnitude of the misconception exhibited by these errors, one wonders how they could have gone undetected in RISA's review process.

The discussion presented here sheds some light on this issue. As I indicated above, a more comprehensive discussion can be found in the article: *Fooled by info-gap decision theory*[1].

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April 26, 2013

[1]See the article *Fooled by info-gap decision theory* at http://www.moshe-online.com/Risk-Analysis-101

1 Introduction

In this discussion I focus on two seemingly properly argued for statements, that were made in two articles published in the ISSUE:

> These two concepts of robustness–min-max and info-gap–are different, motivated by different information available to the analyst. The min-max concept responds to severe uncertainty that nonetheless can be bounded. The info-gap concept responds to severe uncertainty that is unbounded or whose bound is unknown.
>
> Ben-Haim (2012a, p. 1644)

> For small α, searching set $U(\alpha, \tilde{u})$ resembles a local robustness analysis. However, α is allowed to increase so that in the limit the set $U(\alpha, \tilde{u})$ covers the entire parameter space and the analysis becomes one of global robustness. The analysis of a continuum of uncertainty from local to global is one of the novel ways in which info-gap analysis is informative.
>
> Hall et al. (2012, p. 1662)

Clearly, given that RISA is a **peer-reviewed journal** specializing in **risk analysis,** one imagines that (non-expert) readers would treat these two technical statements as unquestionably sound, perhaps even edifying. But, the fact of the matter is that one need not even be an experienced risk analyst to see, at a glance, that these statements are **grossly in error.**

Indeed, those who are versed in Wald's famous maximin paradigm (Wald 1939, 1945, 1950, Rawls 1971, Resnik 1987, French 1988, Sniedovich 2008) would immediately see that the claim that "...The min-max concept responds to severe uncertainty that nonetheless can be bounded ..." is without any foundation because,

- A bounded uncertainty is neither a necessary nor a sufficient condition for a maximin/minimax model to be applicable/valid.

To illustrate, observe that the manifestly **unbounded** minimax model

$$z^* := \min_{-\infty<x<\infty} \max_{-\infty<u<\infty} \left\{x^2 + 2ux - u^2\right\}$$

is a perfectly kosher model.

Clearly then, Ben-Haim's (2012) claim that minimax/maximin models require the uncertainty to be bounded is groundless.

Indeed, since info-gap's robustness model itself is a simple maximin model, how can it possibly be the case that this model can handle an unbounded uncertainty, whereas maximin models cannot?!

Next:

If you are familiar with elementary, standard mathematical notation, and with the definition of info-gap robustness (Ben-Haim 2001, 2006, 2010), namely with:

$$\hat{\alpha}(q, r_c) := \max_{\alpha \geq 0} \left\{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha, \tilde{u})\right\}, \ q \in Q \tag{1}$$

you would immediately conclude that Hall et al.'s (2012) claim is based on an astounding technical error. To wit: this definition clearly states, in no uncertain terms, that the range of admissible values of α is **bounded above** by $\hat{\alpha}(q, r_c)$.

Isn't it therefore clear as a daylight that the definition of info-gap robustness unequivocally asserts that, the admissible values of α are **bounded above** by $\hat{\alpha}(q, \tilde{u})$?

Indeed, if as claimed by Hall et al. (2012), α is **unbounded above,** how is it then that the definition of info-gap robustness **prescribes maximizing** α?

How can one possibly seek to **maximize** the value of α if, as claimed by Hall et al. (2012), it is **unbounded above**?

In short, Hall et al.'s (2012) claim that info-gap decision theory "allows" α to grow, apparently indefinitely, so that at the limit, as $\alpha \rightarrow \infty$, the neighborhood $U(\alpha, \tilde{u})$ covers the entire parameter space to the effect that the analysis becomes one of global robustness, is ..." nothing short of **absurd.**

The question is then:

- How could claims such as these, based as they are on glaring technical errors such as these, possibly be sanctioned by a peer-reviewed journal specializing in **risk analysis**?

In what follows I examine the above quoted claims with the view to shed some light on this question.

But, prior to this, I need to point out that these and similar claims in the literature on info-gap decision theory are part of info-gap proponents' larger objective to convince decision makers, analysts, etc. that info-gap decision theory is **radically different** from all current theories for decision under uncertainty, that it is **novel, powerful,** and so on.

It is important therefore to put the rhetoric exemplified in the above quoted claims in the wider context of the rhetoric in the literature on info-gap decision theory, and to juxtapose it with the hard facts that attest to what this theory in fact is.

The reader is reminded again that this discussion is a shorter version of a more comprehensive article on this subject. The latter is available on my website[1]. This shorter version focuses primarily on the technical errors in Ben-Haim (2012) and Hall et al. (2012) referred to above.

But first let us take a quick look at the theory itself.

2 What is info-gap decision theory?

The rhetoric in the literature on info-gap decision theory strives to impress on the reader that info-gap decision theory is a fairly new, non-probabilistic theory for decision-making under severe uncertainty, that is radically different from all current theories for decision under uncertainty. Indeed, this is how it is presented in the two primary texts on this theory, namely Ben-Haim (2001, 2006).

The fact of the matter is, however, as RISA referees should have known, that the hard facts about this theory tell a different story altogether. I need hardly add that these have been available in the peer-reviewed literature since 2007 (e.g. Sniedovich 2007, 2008, 2010, 2012, 2012a).

Indeed, as these **hard facts** clearly show, not only that this theory does not introduce any new idea or new insight into the modeling, analysis, hence management, of severe uncertainty, its central pillar, namely its robustness model, is a **reinvention** of the concept **radius of stability** (circa 1960) which has been staple fare in many fields for decades. Furthermore, this reinvented model of local robustness, which is claimed, in some info-gap publications, to generalizes the foremost paradigm for decision-making under non-probabilistic uncertainty, namely Wald's **maximin paradigm** (circa 1940), is in fact a very simple, more accurately simplistic, some would say naive, ... maximin model.

To see how this reinvented model of local robustness is adopted by info-gap decision theory, let us consider the generic robustness problem addressed by the theory. This problem involves a *decision variable* $q \in Q$ and a parameter $u \in \mathscr{U}$ whose true value is unknown, indeed is subject to **severe uncertainty.** Thus, $\mathscr{U}$ represents the set of possible/plausible values of u. We shall refer to u as the *uncertainty parameter* and to $\mathscr{U}$ as the *uncertainty space*.

The objective is to determine the **robustness** of decisions against the uncertainty in the true value of u with respect to a **performance requirement** of the form

$$r_c \leq r(q,u) \tag{2}$$

where r_c is a numeric scalar representing a *critical level of performance* and $r(q,u)$ denotes the performance level of decision q given u. View r as a real-valued function on $Q \times \mathscr{U}$.

It is important to realize that

- There are several ways to approach the modeling of robustness in this context.
- Info-gap decision theory takes a **local** approach.

That is, according to info-gap decision theory, the robustness of decision q is a measure of the decision's ability to satisfy the performance constraint for values of u in a **neighborhood** around a **nominal value** of u. This means that to define its measure of robustness, info-gap decision theory brings two additional constructs into play:

$$\tilde{u} := \textbf{nominal value} \text{ of } u. \tag{3}$$

$$U(\alpha,\tilde{u}) := \textbf{neighborhood} \text{ of size } \alpha \geq 0 \text{ around } \tilde{u}. \tag{4}$$

The neighborhoods $U(\alpha,\tilde{u}), \alpha \geq 0$ are assumed to be **nested,** namely $\alpha' < \alpha'' \rightarrow U(\alpha',\tilde{u}) \subseteq U(\alpha'',\tilde{u})$, and $U(0,\tilde{u}) = \{\tilde{u}\}$. Thus, with no loss of generality assume that $\mathscr{U}$ is the smallest set such that $U(\alpha,\tilde{u}) \subseteq \mathscr{U}, \forall \alpha \geq 0$.

Clearly then, the measure that info-gap decision theory deploys to define/assess the robustness of decisions against the uncertainty in the true value of u is not concerned with possible/plausible variations in the value of u over $\mathscr{U}$. Rather, it seeks to determine the robustness of decisions against variations in the value of u in the **neighborhood** of $\tilde{u}$. In practice, this nominal value of u represents a **point estimate** of the true value of u.

To stress this important distinction, let me rephrase it as follows:

- Info-gap decision theory **does not address** this question: how robust is decision q against variations in the value of u over $\mathscr{U}$?
- Info-gap decision theory **address** this question: how robust is decision q to variations in the value of u in the **neighborhood** of the point estimate $\tilde{u}$?

In short, info-gap decision theory (Ben-Haim 2001, 2006, 2010) defines the info-gap robustness of decision $q \in Q$ at $\tilde{u}$ as follows:

▶ DEFINITION 1.

Info-gap's robustness model:

$$\hat{\alpha}(q,r_c) := \max_{\alpha \geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\} \ , \ q \in Q. \tag{5}$$

In words:

- *The info-gap robustness of decision q at $\tilde{u}$, denoted $\hat{\alpha}(q, r_c)$, is equal to the size α of the largest neighborhood $U(\alpha, \tilde{u})$ around $\tilde{u}$ all whose elements satisfy the performance constraint $r_c \leq r(q, u)$.*

The larger $\hat{\alpha}(q, r_c)$, the more (info-gap) robust decision q at $\tilde{u}$.

Pursuant to this definition of robustness, **info-gap's robust-satisficing approach** then **ranks** decisions according to their robustness index: the larger the robustness the better, hence the best (optimal) decision is that whose robustness is the largest (Ben-Haim 2006, 2010, 2012a). Consequently, the optimal decision is obtained by solving the following optimization problem:

▶ DEFINITION 2.

Info-gap's robustness-satisficing decision model:

$$\hat{\alpha}(r_c) := \max_{q \in Q} \ \hat{\alpha}(q, r_c). \tag{6}$$

It is important to keep firmly in mind that info-gap decision theory claims to provide a reliable tool for the treatment/management of **severe** uncertainty and that the severity of the uncertainty is manifested (on this theory) in the following properties:

- The uncertainty space $\mathscr{U}$ can be vast, typically **unbounded**.
- The point estimate $\tilde{u}$ is **poor** and can be substantially **wrong.** It is often a **guess,** sometimes a **wild guess.**
- The uncertainty is **probability-free, likelihood-free, belief-free,** and so on.

This means that any assessment of info-gap decision theory as *a theory for decision-making under severe uncertainty* **must be based** on this understanding of the term "severe uncertainty". And by the same token, any assessment of the rhetoric on info-gap decision theory **must be done** against the definition of info-gap robustness (5) in conjunction with this understanding of the term "severe uncertainty".

With this in mind, let us now examine how the hard facts about the above two models reflect on info-gap decision theory as a whole.

3 The spin free zone

To put across how greatly misleading the rhetoric on info-gap decision theory really is, it is essential to compare its central pillar, namely its robustness model, with other standard models of robustness. Fortunately, to do this, there is no need to embark on a comprehensive literature survey, as all it takes to place info-gap's robustness model in the state of the art, is to examine three run of the mill models of robustness.

3.1 State of the art

Rather than refer to the specific performance requirement defined by (2), namely $r_c \leq r(q, u)$, let us consider a more abstract setup, namely let

$$A(p) := \text{ set of **acceptable** values of } u \text{ associated with decision } q \in Q \tag{7}$$

and consider the performance constraint

$$u \in A(q) \ , \ q \in Q \tag{8}$$

observing that in the case of info-gap decision theory, we can let $A(q) = \{u \in \mathscr{U} : r_c \leq r(q,u)\}$.

The following is a staple measure of local robustness/stability dating back to the 1960s which, since then has been used extensively in many fields of expertise (Wilf 1960, Milne and Reynolds 1962, Hindrichsen and Pritchard 1986a, 1986b, Zlobec 1987, Anderson and Bernfeld 2001).

▶ DEFINITION 3. *The* RADIUS OF STABILITY *of decision q at $\tilde{u}$, denoted $\rho(q,\tilde{u})$, is the size (radius) of the largest neighborhood around $\tilde{u}$ that is contained in $A(q)$. In symbols,*

$$\rho(q,\tilde{u}) := \max_{\alpha\geq 0} \{\alpha : U(\alpha,\tilde{u}) \subseteq A(q)\} \ , \ q \in Q \tag{9}$$

$$= \max_{\alpha\geq 0} \{\alpha : u \in A(q), \forall u \in U(\alpha,\tilde{u})\}. \tag{10}$$

The larger $\rho(q,\tilde{u})$ the more robust q at $\tilde{u}$. ◀

In words:

- The radius of stability of decision q at $\tilde{u}$, denoted $\rho(q,\tilde{u})$, is the radius α of the largest neighborhood $U(\alpha,\tilde{u})$ around $\tilde{u}$ all whose elements are acceptable with respect to decision q.

The following is an intuitive "global counterpart" of the radius of stability model, whose origins also go back to the 1960s (Starr 1963, 1966). It is a measure of **global** robustness that is based on the "size" or "volume" of $A(q)$. Its application therefore relies on the existence of a tractable measure of "size" or "volume" of subsets of $\mathscr{U}$. Let then $SIZE(V)$ denote the "size" or "volume" of set $V \subseteq \mathscr{U}$. For instance, if $\mathscr{U}$ consists of finitely many elements, we can let $SIZE(V) = |V|$ where $|V|$ denotes the cardinality of set V.

▶ DEFINITION 4. *The Size/Volume robustness of decision q is equal to the "size" or "volume" of the set of acceptable values of u associated with it, namely the size or volume of $A(q)$. In symbols,*

$$Rob(q) := SIZE(A(q)) \ , \ q \in Q \tag{11}$$

$$= \max_{V\subseteq\mathscr{U}} \{SIZE(V) : u \in A(q), \forall u \in V)\}. \tag{12}$$

The larger $Rob(q)$ the more robust q. ◀

Note that the difference between this measure of global robustness and radius of stability robustness is that the former seeks the size of the largest acceptable **subset** of $\mathscr{U}$, whereas the latter seeks the size of the largest acceptable *neighborhood* of $\mathscr{U}$ around $\tilde{u}$. The difference between these two measures of robustness is fundamental: while **neighborhood** is an inherently "local" object, **subset** is not.

Next, consider the foremost paradigm for robust-decision making in the face of a non-probabilistic uncertainty namely, Wald's famous maximin paradigm (Wald 1939, 1945, 1959, Rawls 1971, Resnik 1987, French 1988, Sniedovich 2008). It puts forward the following simple rule:

▶ DEFINITION 5. **Wald's maximin rule**
Rank alternatives according to their WORST *outcomes. Hence, select the alternative whose* WORST *outcome is as good as the* WORST *outcomes of the other alternatives.* ◀

Maxinim/minimax models are mathematical transliterations of this rule. They can therefore take various formulations. For our purposes, it is instructive to consider maximin models of the following generic form:

Generic maximin model:

$$z^{\diamond} := \max_{x\in X} \min_{s\in S(x)} \{f(x,s) : con(x,s), \forall s \in S(x)\} \tag{13}$$

where

$$X = \text{ set of } \textit{alternatives} \text{ under consideration.}$$
$$S(x) = \text{ set of } \textit{states} \text{ associated with alternative } x.$$
$$con(x,s) = \text{ list of } \textit{constraints} \text{ imposed on the pairs } (x,s).$$
$$f(x,s) = \textit{payoff/reward} \text{ associated with } (x,s).$$

Recall that in decision-making under uncertainty, the state variable s represents an *uncertainty parameter* whose true value is controlled by *Nature*, rather than the decision maker.

For the purposes of this discussion it suffices to note that Wald's maximin paradigm plays a central role in the areas of decision theory, robust control, robust statistics, robust optimization, and the list goes on. An extensive examination of this paradigm, from the viewpoint of robust decision-making, can be found in Sniedovich (2012e).

That said, given the rhetoric in RISA about info-gap decision theory's extraordinary prowess to deal with **severe** uncertainty, the question naturally arising is whether RISA referees attempted to determine the following:

- Where precisely does info-gap's robust-satisficing approach fit in the state of the art?

The discussion that follows provides some useful clues on this issue.

3.2 Some hard facts

Once the rhetoric on info-gap decision theory is peeled away and the focus is placed squarely on this theory's mathematical models and their mode of operation, the following hard facts are exposed:

- **Fact 1:** Info-gap robustness is a reinvention of the concept **radius of stability** (circa 1960).
- **Fact 2:** Both info-gap robustness model and info-gap robust-satisficing decision model, are simple **maximin models,** more accurately, simple instances of generic models of **Wald's famous maximin paradigm.**
- **Fact 3:** The info-gap robustness of decision q is a measure of the decision's **local** robustness in the neighborhood of $\tilde{u}$. Therefore, the ranking of decisions, hence the choice of an optimal decision, may vary as the value of $\tilde{u}$ is varied over $\mathscr{U}$.
- **Fact 4:** Much as info-gap decision theory's definition of the neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$, allows α to increase indefinitely, **info-gap's robustness model** dictates that **the admissible values of α are bounded above by $\hat{\alpha}(q, \tilde{u})$.**
- **Fact 5:** The info-gap robustness of decision q at $\tilde{u}$ is therefore determined in **complete disregard** for the decision's resilience/vulnerability to deviations in $\tilde{u}$ that are larger than $\hat{\alpha}(q, \tilde{u}) + \varepsilon, \varepsilon > 0$. Namely, the definition of info-gap robustness takes no account whatsoever of values of u outside the neighborhood $U(\alpha^*, \tilde{u}), \alpha^* = \hat{\alpha}(q, \tilde{u}) + \varepsilon, \varepsilon > 0$.
- **Fact 6:** Hence, the literature on info-gap decision theory paints a grossly misleading picture of the theory and its role and place in the state of the art.

More details on these facts can be found in Sniedovich (2007, 2008, 2010, 2012, 2012a) and in the longer version of this article[1]. Here are some of the consequences of these hard facts:

- **Consequence 1:** **Wald's maximin paradigm** offers an incomparably more **general, flexible, hence powerful** framework for the treatment of severe uncertainty than the framework offered by info-gap decision theory.
- **Consequence 2:** **Info-gap's robust-satisficing approach** is an extremely simple, more accurately simplistic, some would say **naive, robust-optimization approach.**
- **Consequence 3:** Because it prescribes an inherently **local** robustness analysis, info-gap decision theory is **utterly unsuitable** for the treatment of a **severe** uncertainty of the type that info-gap decision theory claims to address.
- **Consequence 4:** Given all this, it would clearly be **unwise to accept uncritically** the latest **rhetoric** about info-gap decision theory's prowess.

With this in mind let us now examine some of the latest claims in the ISSUE.

4 The maximin connection

The lengthy discussion (See Appendix A) in Ben-Haim (2012, p. 1643-1644) clearly seeks to persuade that info-gap robustness and min-max robustness are **dissimilar** and, in so doing, it glosses over the fact that . . . info-gap's robustness model is in truth a simple . . . maximin model.

In other words, this lengthy discussion on the alleged differences between info-gap's robustness model and maximin models misrepresents the connection of info-gap's robustness model to Wald's famous maximin paradigm. I might add that it is equally misleading on several other topics. It is important, therefore, to put these assertions aright and to do this consider the following simple question:

- The alleged differences between the info-gap and maximin robustness analyses notwithstanding, how do the two analyses relate to one another?

To see what I am driving at, consider the following analogy:

- The obvious differences between the two real-valued functions g and h defined below notwithstanding, how do these two functions relate to one another?

$$g(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n \ , \ -\infty \leq x \leq \infty, \ n \in \{1, 2, 3, \dots\} \quad (14)$$

$$h(y) = (y - A)(y - B) \ , \ -\infty \leq y \leq \infty \quad (15)$$

where all the coefficients are numeric scalars, and n is regarded as a parameter?

The answer is crystal clear:

- The obvious differences between these two functions notwithstanding, both are intimately connected because:
  - Both are **polynomials.**
  - h is a simple **instance** of g.

In other words, whatever the differences between g and h, these are differences between a **prototype** (g) and one of its (very) simple **instances** (h).

As indicated by Fact 2, this intimate relationship is totally analogous to the kinship of maximin models of robustness and info-gap's robustness model:

- Notwithstanding the obvious differences between these models, they are intimately related in the sense that:
  - They are based on a **worst-case approach** to uncertainty.
  - Info-gap robustness model is a simple **instance** of generic **maximin** models.

Thus:

▶ THEOREM 1. *Info-gap's robustness model (5) and info-gap's robust-satisficing decision model (6) are simple maximin models. Indeed, both are simple instances of the generic maximin model defined by (13).*

PROOF. See Appendix B. ◀

In other words, whatever the differences between generic maximin models and info-gap's robustness model, these are differences between a **prototype** concept (maximin) and one of its many **instances,** in this case the (very) simple info-gap robustness model.

No amount of rhetoric can explain this fact away!

And no amount of rhetoric can explain away the fact that the simple robustness model that info-gap decision theory calls "info-gap robustness", is a reinvented version (a clone) of a well established model that is known universally as **radius of stability**. For the record then:

▶ THEOREM 2. *Info-gap's robustness model (5) is a radius of stability model. Indeed, it is a simple instance of the generic radius of stability model defined in (10).*

PROOF. By inspection, info-gap's robustness model (5) is the instance of the radius of stability model (10) corresponding to the set $A(q) = \{u \in \mathscr{U} : r_c \leq r(q,u)\}$. ◀

There is not a single reference to this critical connection in Ben-Haim (2001, 2006, 2010, 2012, 2012a), or in Hall et al. 2012) or in any other peer-reviewed article advocating the use of info-gap decision theory.

5 Optimizing over unbounded domains

The misguided claim in Ben-Haim (2012a)

> The min-max concept responds to severe uncertainty that nonetheless can be bounded.
>
> Ben-Haim (2012a, p. 1644)

does not belong in a peer-reviewed article.

To see why, simply go back to your introductory undergraduate college/university textbooks. In them you will find numerous **counterexamples** to this absurd claim. So, to bring out the absurd in it, let us examine this claim by way of relating a perfectly true story to two hypothetical stories.

5.1 A highly hypothetical story

Suppose that an ARTICLE published in RISA were to advance the following (manifestly erroneous) proposition.

▶ CLAIM 1. *A real-valued function cannot attain a global optimum on an unbounded domain. That is, let f be a real-valued function on some set X and consider the optimization problem*

$$z^* := \operatorname*{opt}_{x \in X} f(x). \tag{16}$$

Then for this problem to possess a global optimum, set X must be bounded.

PROOF. With no loss of generality assume that $\text{opt} = \max$, and let x be any element of X. Since X is unbounded, there exists an $x' \in X$ such that $f(x') > f(x)$. Hence, there is no $x^* \in X$ such that $f(x^*) \geq f(x), \forall x \in X$, implying that function f does not attain a global maximum on X. ◀

One imagines that, sooner rather than later, the EDITOR would have been flooded with LETTERS arguing that this claim is demonstrably false, its proof showing a fundamental misunderstanding of the concept "optimum", the basic properties of real-valued functions, and so on.

The LETTERS would have advised the EDITOR that the "proof" is in error because the fact that X is unbounded does not imply that for any given $x \in X$ there is an $x' \in X$ such that $f(x') > f(x)$. For instance, the LETTERS might have advised the EDITOR that in the case of $X = (-\infty, \infty)$ and $f(x) = 1 - x^2$, for $x = 0$ there is no $x' \in X$ such that $f(x') > f(x)$, hence by definition, f attains a global maximum on X at $x^* = 0$.

There seems to be little point in speculating on what the EDITOR would have done in this case. One thing is clear, though. Steps would have been taken to correct the ARTICLE in question.

Remark

I refer to the above case as "highly hypothetical" because it is most unlikely that an article propounding such a fundamental error would have been submitted to a journal such as RISA. It is even more unlikely that such a fundamental error would not have been detected immediately in the peer-review process.

Next, let us consider a related case that is less hypothetical.

5.2 A slightly hypothetical story

Suppose that an ARTICLE published in RISA were to make the following claim.

▶ CLAIM 2. *Minimax models require their uncertainty parameter to be bounded. For instance, consider the following simple minimax problem*

$$z^* := \min_{x \in X} \max_{u \in \mathscr{U}} f(x, u) \tag{17}$$

observing that x denotes the decision variable and u denotes the uncertainty parameter. Then for this minimax problem to possess an optimal solution, set $\mathscr{U}$ must be bounded.

PROOF. Assume that set $\mathscr{U}$ is unbounded and let x be any element of X. Since $\mathscr{U}$ is unbounded, for any given $u \in \mathscr{U}$ there exits a $u' \in \mathscr{U}$ such that $f(x, u') > f(x, u)$. Hence, there is no $u^* \in \mathscr{U}$ such that $f(x, u^*) \geq f(x, u), \forall u \in \mathscr{U}$. This implies that the (inner) $\max$ in (17) is not attained. Consequently, the above minimax problem does not have an optimal solution. ◀

Clearly, this is a slight variation on the preceding highly hypothetical case. The only difference between the two is that in this case the error might be less obvious due to the distraction caused by the outer $\min$

operation. But the fundamental technical error is identical in both cases. This is the groundless premise that the domain of a real-valued function must be bounded for the function to attain a global optimum on the domain. The fact of the matter is of course that having a bounded domain is **neither** a necessary **nor** a sufficient condition for a real-valued function to attain a global optimum.

One would have expected, therefore, the EDITOR to be flooded with LETTERS pointing this out.

5.3 A perfectly true story

The case that is of concern to us here is featured in an article that was published in RISA, more specifically in the ISSUE.

▶ CLAIM 3.

> *The* **min-max** *concept responds to severe uncertainty that nonetheless can be* **bounded.**
>
> *Ben-Haim (2012, p. 1644; emphasis added)*

It is immediately clear that this claim makes the same argument as CLAIM 2. It can therefore be dismissed outright as erroneous. Here is a simple counterexample to this misguided claim.

▶ Counterexample

Consider the following simple, unbounded minimax problem:

$$z^\circ := \min_{-\infty<x<\infty} \max_{-\infty<u<\infty} \{x^2 + 2xu - u^2\} \tag{18}$$

observing that x denotes the decision variable and u denotes the uncertainty parameter.

A quick back-of-the-envelope analysis shows that this problem has a unique solution, namely $(x^\circ, u^\circ) = (0, 0)$, yielding $z^\circ = 0$. ◀

The EDITOR should therefore be expected to be flooded with LETTERS pointing this out, and the READERSHIP of the journal is certainly entitled to expect the EDITOR to take steps to set this aright.

What is so remarkable about this perfectly true story is the failure in RISA's peer-review process to diagnose this claim for what it is, hence to appreciate its consequences for the entire analysis in Ben-Haim (2012a). Not only should it have been recognized immediately that this is an unfounded, indeed profoundly mistaken claim, it should have been appreciated that this fact pulls the rug out from under the main assertions put forward in Ben-Haim's (2012a) analysis on the relationship between info-gap decision theory and Wald's maximin paradigm.

6 Global vs local robustness

As explained and illustrated clearly in Sniedovich (2007, 2010, 2012, 2012b), Hayes et al. (2013), Schapaugh and Tyre (2013), and in the ISSUE itself (Sniedovich 2012a), info-gap robustness is a measure of **local** robustness. So, how are we to explain this remarkable claim (emphasis added)?

> The analysis of a continuum of uncertainty **from local to global** is one of the novel ways in which info-gap analysis is informative.
>
> Hall et al. (2012, p. 1662)

I address the reference to the alleged novel way later. For now, I focus on the local vs global robustness issue.

Clarifying the question of local vs global robustness is important for a number of reasons. In our case clarifying this issue is central to exposing the hard facts about info-gap decision theory and the rhetoric that covers them up. Observe then that proponents of info-gap decision theory repeatedly state that one of the distinguishing features of the theory, a feature which as they claim, invests it with the extraordinary ability to deal reliably with extreme events, catastrophes, shocks, and even ... unknown unknowns, is that it allows the uncertainty to be **unbounded.**

It is also important to note that the claim that the uncertainty is allowed to be **unbounded** is also invoked by info-gap proponents to argue that info-gap's robustness analysis therefore cannot possibly be local in nature. Indeed, as info-gap proponents see it, allowing the uncertainty to be **unbounded,** enables info-gap's robustness analysis to be a global robustness analysis thereby rendering it a reliable tool for dealing with a likelihood-free, non-probabilistic uncertainty.

But:

The fact that info-gap robustness is inherently local in nature is so manifestly obvious that one cannot help wondering what "technical" arguments can possibly be offered to show/prove that info-gap robustness is not local after all. Consider then the technical explanation that Hall et al.'s (2012) came up with (emphasis added):

> **For small α, searching set $U(\alpha,\tilde{u})$ resembles a local robustness analysis. However, α is allowed to increase so that in the limit the set $U(\alpha,\tilde{u})$ covers the entire parameter space and the analysis becomes one of global robustness.** The analysis of a continuum of uncertainty from local to global is one of the ways in which info-gap analysis is informative.
>
> Hall et al. (2012, p. 1662)

This "technical explanation" clearly leaves one speechless.

Because, a quick scan of info-gap decision theory's definition of robustness makes it **abundantly clear** that the robustness analysis conducted to determine the info-gap robustness of a decision most definitely **does not allow** α to increase indefinitely. This is so because an upper bound is imposed on the admissible values of α by none other than this ... **definition of info-gap robustness.** Hence,

▶ THEOREM 3. *According to info-gap decision theory, in the context of determining the info-gap robustness of decision q at $\tilde{u}$, the admissible values of α are bounded above by $\hat{\alpha}(q,r_c)$. Hence, the domain that effectively determines (in the robustness analysis) the info-gap robustness of decision q at $\tilde{u}$ is the neighborhood $U(\alpha^*,\tilde{u})$, where $\alpha^*=\hat{\alpha}(q,\tilde{u})+\varepsilon$ and $\varepsilon>0$ can be arbitrarily small.*

PROOF. This follows immediately from the definition of $\hat{\alpha}(q,r_c)$, namely

$$\hat{\alpha}(q,r_c) := \max_{\alpha\geq 0}\left\{\alpha : r_c \leq r(q,u), \forall u\in U(\alpha,\tilde{u})\right\}\ ,\ q\in Q \tag{5}$$

recalling that the sets $U(\alpha,\tilde{u}),\alpha\geq 0$ are nested. ◀

One wonders therefore how this definition can so badly be misread to lead one to conclude that info-gap's robustness analysis of decision q at $\tilde{u}$ "allows" the parameter α to increase indefinitely? Surely, if this were so, what would be the point in seeking to **maximize** the value of α?!

All this goes to show that the claim:

> α is allowed to increase so that in the limit the set $U(\alpha, \tilde{u})$ covers the entire parameter space and the analysis becomes one of global robustness.
>
> Hall et al. (2012, p. 1662)

is not merely a "technical" error. It is a profound conceptual error about the very **definition of info-gap robustness.**

For the record then,

▶ COROLLARY 1. *The range of admissible values of α associated with decision q at $\tilde{u}$ is determined by the definition of info-gap robustness and is therefore equal to* $[0, \hat{\alpha}(q, \tilde{u})]$. ◀

▶ COROLLARY 2. *Info-gap decision theory allows the range of admissible values of α associated with decision q, namely the interval* $[0, \hat{\alpha}(q, \tilde{u})]$*, to be arbitrarily small. Indeed, the theory allows $\hat{\alpha}(q, \tilde{u})$ to be equal to 0.* ◀

No amount of rhetoric can explain these facts away.

To illustrate COROLLARY 2, consider the case where for some decision $q' \in Q$ we have $r_c \leq r(q', \tilde{u})$ and $r_c > r(q', u)$ for all $u \in \mathscr{U} \setminus \{\tilde{u}\}$. Then clearly $\hat{\alpha}(q, \tilde{u}) = 0$ and therefore the range of admissible values of α in this case is $[0, 0] = \{0\}$.

To repeat, no amount of rhetoric can explain this fact away.

More details on Hall et al.'s (2012) blunder can be found in Sniedovich (2012d). For now it suffice to note that this blunder is apparently due to a misinterpretation of the fact that, in certain situations, one can conduct a **parametric analysis** of $\hat{\alpha}(q, r_c)$ with respect to r_c, where for a sufficiently small r_c info-gap robustness namely, α becomes unbounded. In other words, Hall et al. (2012) apparently misunderstand how the following two completely different facts relate to one another:

- The fact that info-gap's robustness analysis seeks decisions that are **locally** robust.
- The fact is that info-gap robustness of decisions is **non-increasing** with r_c, hence the performance constraint $r_c \leq r(q, u)$ can be relaxed (by **decreasing** the value of r_c) to the effect that the constraint becomes **superfluous** (namely $r_c \leq r(q, u)$ for all $q \in Q$ and for all $u \in \mathscr{U}$). This way, the info-gap robustness of decisions can be "forcibly" becomes **unbounded** through this decrease in the value of r_c.

That said, it should be pointed out that the fact that, under certain conditions, the info-gap robustness of decisions can be unbounded does not imply that info-gap robustness is not a measure of local robustness. By the same token that it does not follow that if, under certain conditions, an optimization method generates global optima, this method is a method of global optimization. Indeed, as is well known that, under certain conditions, methods of local optimization can generate global optima.

The qualification "under certain conditions" is designed to remind RISA referees that not always is it the case that the value of r_c is "allowed" to vary, let alone vary to such an extent that the constraint $r_c \leq r(q, u)$ is rendered redundant. Indeed, there are cases where the value of r_c is fixed to the effect that it is "not allowed" to vary at all (e.g. it is determined by natural laws, or by government regulations, etc.). Likewise, there are cases where the value of r_c is "allowed" to vary only within a narrow range. Indeed, as indicated by the Father of info-gap decision theory in RISA:

> Performance requirements may originate in various ways: by legislation, by administrative fiat, by public debate and collective decision making, and so on. Furthermore, multiple constraints of various sorts may be imposed, such as cost and safety constraints, engendering trade-offs.
>
> Ben-Haim (2012, p. 1328)

In sum, the “secret weapon” that, according to Hall et al. (2012), empowers info-gap’s robustness model to perform an “analysis of a continuum of uncertainty from local to global” comes down to . . . **a relaxation of the performance requirement to a degree that it is rendered irrelevant**. Thus “global” robustness is attained when the performance requirement is . . . **redundant!** Or in other word, Hall et al.’s (2012) argument boils down to this:

- In cases where the **robustness constraint is redundant,** info-gap’s measure of local robustness becomes global.

A novel and informative approach to global robustness indeed!

More on this can be found in Sniedovich (2012d).

Given the on-going effort in the info-gap literature to deny, or explain away, the fact that info-gap robustness is local, it is important to have a clear picture of the incontrovertible “localness” of info-gap robustness. Consider then the following immediate implication of THEOREM 3:

▶ COROLLARY 3. *The info-gap robustness of decision q at $\tilde{u}$ is determined in total disregard of the performance of decision q for values of u that are outside the neighborhood $U(\alpha^*, \tilde{u})$ where $\alpha^* = \hat{\alpha}(q, \tilde{u}) + \varepsilon$ and $\varepsilon > 0$ can be arbitrarily small.* ◀

This suggests the following:

▶ DEFINITION 6. *We refer to the neighborhood*

$$ED(q, \tilde{u}, \varepsilon) := U(\alpha^*, \tilde{u}) \ , \ \alpha^* = \hat{\alpha}(q, \tilde{u}) + \varepsilon \ , \ (\varepsilon > 0 \text{ and arbitrarily small}) \tag{19}$$

as the EFFECTIVE DOMAIN *of info-gap’s robustness analysis of decision q at $\tilde{u}$. In contrast, we refer to the set*

$$NML(q, \tilde{u}, \varepsilon) := \mathscr{U} \setminus ED(q, \tilde{u}, \varepsilon) \tag{20}$$

as the NO MAN’S LAND *of decision q at $\tilde{u}$.*

Observe that the info-gap robustness of decision q at $\tilde{u}$ takes no account of the performance levels of decision q for values of u in $NML(q, \tilde{u}, \varepsilon)$. ◀

The bottom line is this: Info-gap robustness is a measure of **local** robustness *par excellence* because:

- Info-gap decision theory **allows** the effective domain of its robustness analysis to be **exceedingly small** relative to the uncertainty space $\mathscr{U}$, thus allowing its robustness analysis to ignore most of the uncertainty space under consideration.
- The effective domains are **neighborhoods** in $\mathscr{U}$.

That is to say, the combined effect of these two factors is that the info-gap robustness of a decision is a measure of the decision’s resilience/vulnerability to variations in the value of u in the neighborhood of $\tilde{u}$ specified by the effective domain of the decision at $\tilde{u}$, where this neighborhood is allowed to be arbitrarily small. This means that there is no assurance that the info-gap robustness of a decision adequately represents the decision’s ability to satisfy the performance constraint as u varies over the uncertainty space $\mathscr{U}$.

It is important to appreciate the implications of these facts for info-gap decision theory’s performance as a . . . decision theory which comes down to a recipe for **ranking decisions** according to their robustness index and selecting the **best** from among them.

Recall then that info-gap's robust-satisficing approach to the ranking of decisions is that the larger the robustness the better. Hence, the best (optimal) decision is that whose info-gap robustness is the largest. However, given what we just saw, it is clear that info-gap's robust-satisficing approach does not seek decisions that are robust against variations in the value of u over the uncertainty space $\mathscr{U}$, but rather decisions that are robust against variations in the value of u in the **neighborhood** of $\tilde{u}$. And since the theory allows this neighborhood to be arbitrarily small (relative to $\mathscr{U}$) the implication is that the results yielded by it would typically have a limited, namely **local**, validity.

6.1 Example

Consider the case where $Q = \{q', q''\}$, $\mathscr{U} = \mathbb{R}^3$, $\tilde{u} = (0,0,0)$, the neighborhood $U(\alpha, \tilde{u})$ is the Euclidean ball of radius α centered at $\tilde{u}$, and the two decisions under consideration have the following properties:

- Decision q' **satisfies** the performance constraint **everywhere** on $\mathscr{U}$ except at points in $\mathscr{U}$ whose distance from $\tilde{u}$ is in the open interval (0.0001,0.0002).
- Decision q'' **violates** the performance constraint **everywhere** on $\mathscr{U}$ except at points in the neighborhood $U(100, \tilde{u})$ which is minuscule relative to $\mathscr{U}$.

Note that, according to info-gap decision theory, $\hat{\alpha}(q', \tilde{u}) = 0.0001$ and $\hat{\alpha}(q'', \tilde{u}) = 100$, hence decision q'' is more info-gap robust than decision q'. Clearly, this preference does not reflect the extent of the decisions' ability to cope with variations in the value of u over $\mathscr{U}$. Rather, it reflects the decisions' robustness in the neighborhood of $\tilde{u} = (0,0,0)$. Indeed, should the info-gap robustness of these decisions be measured in the neighborhood of say $\tilde{u}' = (1000, 1000, 1000)$, it would turn out that decision q' is far more info-gap robust than decision q''. ◀

The question therefore is:

- How can a local analysis in the neighborhood of a wild guess possibly be regarded as a reliable tool for the management of a severe uncertainty of the type addressed by info-gap decision theory?

My point is that this question should have been raised by RISA referees a long time ago! I address it in the next section.

7 Voodoo decision-making

Epithets such as *voodoo economics, voodoo science, voodoo mathematics, voodoo statistics,* and so on, are commonly used, in the respective areas of expertise, to designate questionable theories, methods, paradigms and so on. I use the terms *voodoo decision-making* and *voodoo decision theory* in the same vein (e.g. Sniedovich 2010, 2012, 2012a, 2012b) to designate the claims, in the literature on info-gap decision theory, that this theory provides a reliable tool for the modeling, analysis and management of severe uncertainty where, according to Ben-Haim (2001, 2006, 2010), the severity of the uncertainty is manifested in these characteristics:

- The uncertainty space $\mathscr{U}$ is allowed to be vast, typically **unbounded**.
- The point estimate $\tilde{u}$ is **poor** and can be substantially **wrong.** It is often a **guess,** sometimes a **wild guess.**
- The uncertainty is **probability-free, likelihood-free, belief-free,** and so on.

The following discussion explains and justifies my position.

The fact that must be kept firmly in mind is that info-gap decision theory imposes no restrictions on the range of the effective domains $ED(q,\tilde{u},\varepsilon)$, $q \in Q$ over which the robestness analysis is conducted. That is, it is of no concern whatsoever in an info-gap robustness analysis how small (or, for that matter large) the effective domains $ED(q,\tilde{u},\varepsilon), q \in Q$ are. Consequently, the ranking of decisions according to their info-gap robustness $\hat{\alpha}(q,\tilde{u})$ is determined irrespective of what the values $ED(q,\tilde{u},\varepsilon), q \in Q$ are.

But consider what this means. Since these domains are allowed to be arbitrarily small, it follows that info-gap decision theory deems "reliable" robustness analyses that are conducted on arbitrarily small effective domains, namely on minutely small neighborhoods of a potentially vast uncertainty space $\mathscr{U}$.

The absurd of this position becomes clear when you consider that in view of the severe uncertainty postulated by info-gap decision theory, **to begin with** there is no reason to believe that the results of its analyses are "reliable". In other words, there is no reason to believe that the ranking based on the info-gap robustness of decisions is a good approximation of a ranking that is based on a robustness analysis over the entire uncertainty space. Indeed, it is easy to construct examples where local and global robustness analyses generate completely different rankings (see Sniedovich 2010, 2012). Let alone then in cases where the effective domains are minute!

To counter any protestation that my dwelling on minutely small neighborhoods as a basis for my argument is "unfair" to the theory, I call attention to the following fact.

According to Ben-Haim (2001, 2006, 2010), the uncertainty space of an info-gap uncertainty model is **typically unbounded**. This means that **typically** the effective domains $ED(q,\tilde{u},\varepsilon), q \in Q$ are not only small but **infinitesimally small,** in relation to $\mathscr{U}$.

And yet Ben-Haim (2001, 2006, 2010, 2012a) and Hall et al. (2012) propose this theory, which typically prescribes a local robustness analysis over an infinitesimally small neighborhood of a wild guess, as a reliable tool for the modeling, analysis and management of a non-probabilistic, unbounded uncertainty!

Figure 1 illustrates the absurd of this proposition—an absurd that justifiably earns info-gap decision theory the title "voodoo decision theory *par excellence*".

What is so disturbing in all this is that the rhetoric that is used to justify the claims denying that info-gap decision theory is a theory of local robustness continues to be accepted by RISA referees. And to illustrate what I have in mind, consider the following statement:

> If the robustness is not large, and especially if the robustness is small, then confidence is not warranted. If the robustness is small then confidence is warranted only "locally," near the models, while if the robustness is large then confidence is warranted over a wide domain of deviation from the models. Info-gap theory uses the analyst's models, but this does not makes it a "local" theory of robustness.
>
> Ben-Haim (2012a, p. 1644)

It is important to point out therefore that:

- Indeed, it is not the fact that "info-gap theory uses the analyst's models" (read: estimates) nor that these models (read: estimates) can be poor, even wild guesses, that makes it a theory of local robustness and a voodoo decision theory at that.
- Rather: what makes info-gap decision theory a theory of local robustness *par excellence* is its **definition of robustness** which is known universally as *radius of stability.* It is the type of robustness analysis prescribed by this definition of robustness that makes info-gap a theory of local robustness *par excellence*.

- That is, it is the fact that info-gap's robustness model typically conducts its robustness analysis on a tiny neighborhood of an unbounded uncertainty space around a poor point estimate, **to the exclusion of** the area outside this tiny neighborhood, that makes info-gap a theory of local robustness *par excellence.*
- The fact that the info-gap robustness of decisions can be large does not alter the fact that info-gap decision theory is a theory of local robustness. Because, to reiterate, it is its **definition of robustness** that determines this fact.

The bottom line is this.

Proponents of info-gap decision theory cannot have it both ways:

- If they accept the established definition of info-gap robustness, then they have to accept that info-gap decision theory is a theory of local robustness and that it is therefore an unsuitable measure of robustness for the treatment of a severe uncertainty of the type that info-gap decision theory claims to address.
- If they assume that the uncertainty under consideration is as severe as that postulated by the theory, then they ought to use a measure of robustness that is capable of taking on such an uncertainty. The radius of stability model that info-gap decision theory prescribes for this purpose is clearly unsuitable for the treatment of a severe uncertainty of the type that is postulated by the theory.

My point is that the trouble with info-gap decision theory is not in the fact that it is a theory of local robustness. Obviously, models of local robustness of the *radius of stability* type have been staple fare in various areas of expertise for decades. The trouble with info-gap decision theory is in misapplication of this model: in its prescribing the local (radius of stability) model to situations where models of local robustness are utterly unsuitable.

And to conclude this section, I want to revisit the main reason for my ascribing the epithet "voodoo" to info-gap decision theory.

To be able to bring things into sharp focus it would be best to clarify this issue in the context of an anonymous decision theory call it THEORY.

Assume then that THEORY prescribes determining the robustness of decision $q' \in Q$ against a severe uncertainty in the true value of $u \in \mathscr{U}$ by means of an analysis of the performance of q' at values of u in the set represented by the white area shown in Figure 1 while **completely ignoring** the performance of q' at values of $u \in \mathscr{U}$ in the black area in the figure, where $\mathscr{U}$ represents the set of possible/plausible values of u.

Next, suppose that in my capacity as a consultant, I were to propose that you adopt THEORY for the purpose of managing a severe uncertainty of the type stipulated by info-gap decision theory. Surely, the least that you would expect of me is that I justify the use of THEORY. Namely, that I outline the reason(s) for THEORY ignoring the values of u in the huge black area of $\mathscr{U}$. But what is more, that I make a good case for this fact showing that it makes good sense, or that it is imperative, or that it is expeditious, and so on. For one can envisage situations where, due to special circumstances, it might make sense for the robustness analysis to ignore all values of $u \in \mathscr{U}$, except for those in a tiny neighborhood in $\mathscr{U}$.

Clearly, what I am driving at is that an explanation, indeed a sound justification, for such a proposition are not only expected, they are required.

And to go back to info-gap decision theory. The trouble with it is not only in the fact that it prescribes a robustness analysis that ignores the bulk of the uncertainty space under consideration, except for a minuscule neighborhood in it. It is also in the fact that it makes this radical proposition without offering the slightest explanation and justification for it.

These are the reasons for my dubbing info-gap decision theory a "voodoo decision theory par excellence".

There are, of course, other ways to describe decision makers and/or decision theories that ignore the full range of possible variation of the uncertainty parameter. For instance, Ben-Tal et al. (2009b, p. 926) argue eloquently that models that restrict the analysis to the "normal range" of the parameter of interest rather than seek to explore the entire uncertainty space "... represent a somewhat 'irresponsible' decision maker ...."

What is important here is not necessarily how such theories are branded "voodoo decision theories" or "irresponsible decision theories" or whatever. What is important is that such theories be exposed for what they are. Namely, it is important to make the methods that such theories propose abundantly clear, and if an incongruity exists between these methods and the narrative describing them, then it is imperative to make this fact known.

8 Conclusions

The challenge of coping with uncertainty is of course the lot of all humans:

> The most deplorable and tragic of all human weaknesses is undoubtedly our total incapacity for seeing into the future, which is in sharp contrast to so many of our gifts and our knowledge.
>
> Andric (1994, p. 266)

Still, analysts—not to mention decision and policy makers—are burdened with special challenges, hence the ongoing effort to formulate approaches for dealing with uncertainty.

Concepts such as "robustness" and "stability" have long figured prominently in this effort. Indeed, the concept "robustness" has been used (for more than four decade) as a tool of thought (for instance in the broad area of decision making) in the analysis, formulation, and even the solution of problems subject to a non-probabilistic uncertainty. Likewise, the concept "stability" has (even longer) been the foremost tool in the analysis and formulation of "local stability" in various areas of expertise for instance, applied mathematics, engineering, economics. The literature on this topics is enormous.

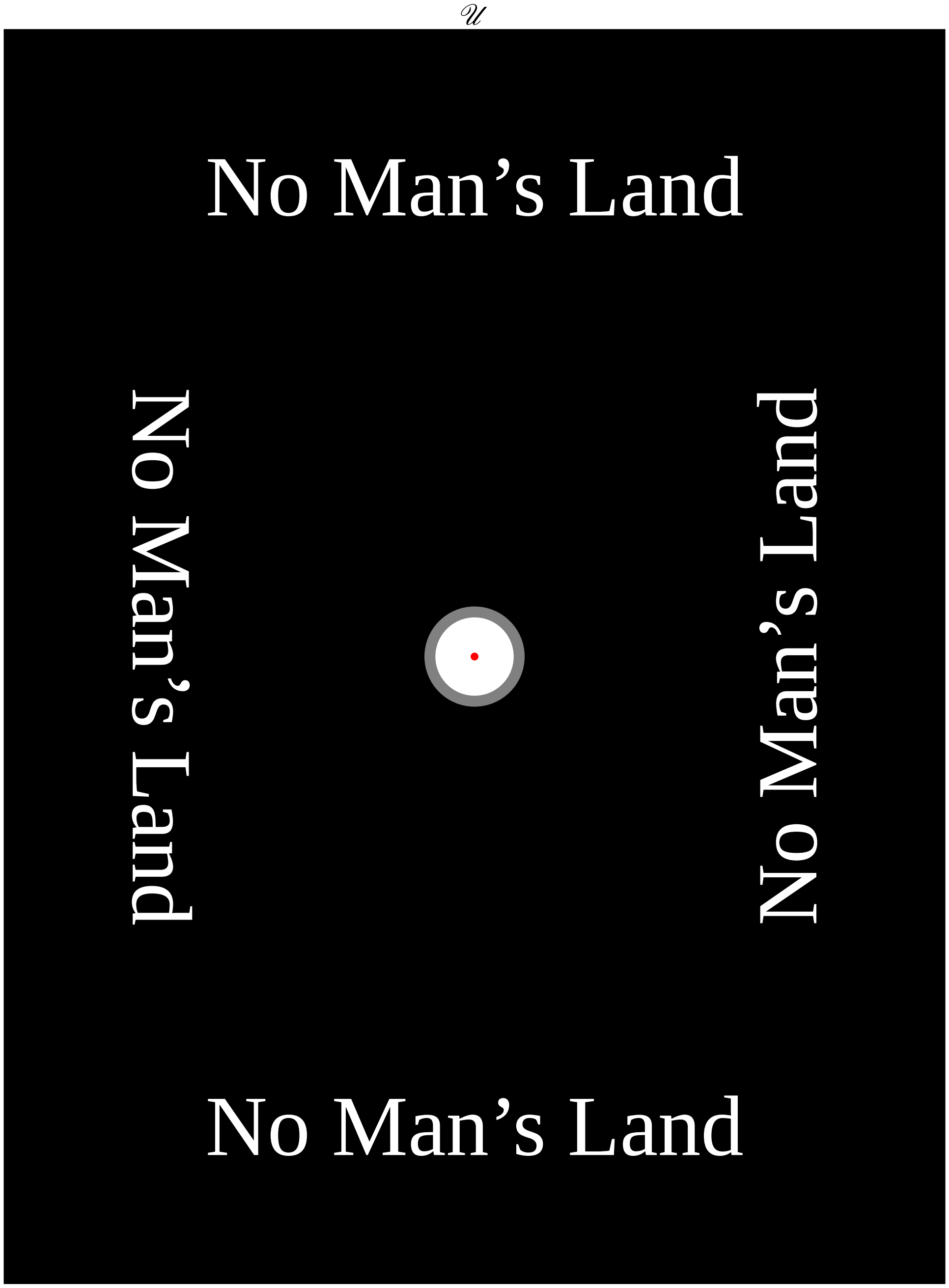
This means that it should not be all that difficult to identify a misleading, unsubstantiated rhetoric about the role of "robustness" in the effort of coping (effectively) with severe uncertainty. And by the same token, it should not be all that difficult to identify models that are a reinvention of the old warhorse "radius of stability".

Thus, to be blunt about it.

Egregious technical and a misleading rhetoric of the kind found in the statements:

> These two concepts of robustness–min-max and info-gap–are different, motivated by different information available to the analyst. The min-max concept responds to severe uncertainty that nonetheless can be bounded. The info-gap concept responds to severe uncertainty that is unbounded or whose bound is unknown.
>
> Ben-Haim (2012a, p. 1644)

> For small α, searching set $U(\alpha,\tilde{u})$ resembles a local robustness analysis. However, α is allowed to increase so that in the limit the set $U(\alpha,\tilde{u})$ covers the entire parameter space and the analysis becomes one of global robustness. The analysis of a continuum of uncertainty from local to global is one of the novel ways in which info-gap analysis is informative.
>
> Hall et al. (2012, p. 1662)



The large rectangle represents the uncertainty space $\mathscr{U}$, the red dot in the center of the white circle represents the point estimate $\tilde{u}$, the white circle represents the neighborhood $U(\alpha^*, \tilde{u}), \alpha^* = \hat{\alpha}(q, \tilde{u})$, and the gray area represents that part of neighborhood $U(\alpha', \tilde{u}), \alpha' = \hat{\alpha}(q, \tilde{u}) + \varepsilon$ that is not covered by $U(\alpha^*, \tilde{u})$. The large black area represents the *No Man's Land* $NML(q, \tilde{u}, \varepsilon)$. This makes vivid that info-gap's robustness analysis of the decision under consideration completely ignores the values of the uncertainty parameter in the black area. To fully appreciate the implications of this fact keep in mind that info-gap decision theory's uncertainty space $\mathscr{U}$ is typically **unbounded**...

Figure 1: A profile of a voodoo decision theory: the *No Man's Land* effect.

have no place in a peer-reviewed journal.

As indicated in Sniedovich (2012c), adopting the approach that the rhetoric on info-gap robust-satisficing decision-making proposes, not only will not advance us in our quest for methods for dealing with severe uncertainty, it will take us back to the 1960s. It will take us back to the early days of *robust optimization,* to the era predating the new development in this area of expertise. This can do only one thing: hinder followers of info-gap robust-satisficing decision-making from benefiting from the enormous progress that has been done, in this area, in the past fifty years or so.

I therefore recommend that not only referees and editors, but all readers, be more vigilant when reading publications on info-gap decision theory. Since this theory's two core models are, simple immediately intelligible mathematical models, it should be easy to put the rhetoric about this theory to the test. All one needs to do to this end is to check whether the narrative describing this theory and its capabilities is consistent with these models' mathematical formulation, their mode of operation and their scope. It goes without saying that this presupposes more than just a fleeting acquaintance with info-gap decision theory.

I remind the reader that a more comprehensive discussion on this topic is available on my website[1].

Appendices

A An example of the latest info-gap rhetoric recently published in *Risk Analysis*

When reading the following lengthy misleading rhetoric on the alleged differences between info-gap robustness and maximin robustness, keep in mind that:

► THEOREM 1. *Info-gap's robustness model (5) and info-gap's robust-satisficing decision model (6) are simple maximin models. Indeed, they are simple instances of the generic maximin model defined by (13).*

PROOF. See Appendix B. ◄

In this discussion I focus only on the last paragraph of the quoted text. A more comprehensive analysis of the misleading rhetoric contained in this text can be found in Sniedovich (2012e).

> **3.4. Robustness and Worst Cases: Two Approaches**
>
> There are many types of risk analysis partly because ignorance and uncertainty come in many forms. Probabilistic uncertainty induces probabilistic risk analysis, while starker uncertainty—for instance, ignorance of relevant probability distributions—engenders other analyses of risk.
>
> A widely occurring operational distinction between risk analyses hinges on whether or not meaningful worst cases can be identified. When one can plausibly specify the worst events that can occur (and presuming we don't know probability distributions), then one might justifiably try to ameliorate these worst contingencies. This can be done in many different ways, and we will refer to this type of strategy as min-max analysis: minimizing the maximum damage.
>
> The ability to implement a min-max analysis depends on identifying meaningful worst contingencies. This is feasible in many situations. The concept of a "meaningful worst case" depends on knowledge and judgment that may be within the risk analyst's competence. However, it is not usually sufficient to specify a worst case in some formal or abstract sense, such as the set of all contingencies that are consistent with the laws of science. A min-max analysis based on such an inclusive formulation may be uselessly overconservative. Min-max analysis is most useful when the analyst is able to avoid vacuous specification of worst cases. However, when information is really scarce, for instance, when processes are poorly understood or changing, then even typical cases cannot be reliably identified. It may then be impossible to meaningfully specify the boundary between extreme but possible occurrences, and the impossible or negligible.
>
> Nonetheless, even when worst cases cannot be meaningfully specified, the analyst still has data, understanding, and mathematical representations: models in the broad sense that we are using that term. It is simply that the analyst cannot responsibly specify the magnitude of error of these models. For instance, we have many models for long-range climate change, but the earnest scientific disputes over these models may preclude the ability to confidently bound the errors. Or, introducing a new species to an ecosystem, either deliberately as an genetically modified organism or inadvertently by invasion, may alter the ecosystem dynamics in unknown ways.
>
> In such situations one can still formulate and implement a robustness analysis. Info-gap theory has been developed precisely for the task. Let's discuss min-max and info-gap concepts of robustness.

> The min-max concept of robustness responds to the question: How bad is the worst case? This is valuable information for the risk analyst and decisionmaker because if the worst case—after amelioration by a min-max analysis—is tolerable, then one can reasonably say that the system is robust to uncertainty.
>
> The info-gap concept of robustness responds to a different question: How wrong can the models be and still guarantee that the outcome is acceptable? This is useful for the risk analyst and decisionmaker because if the models can err enormously without preventing acceptable outcomes, then one can reasonably say that the system is robust to uncertainty.
>
> These two concepts of robustness—min-max and info-gap—are different, motivated by different information available to the analyst. The min-max concept responds to severe uncertainty that nonetheless can be bounded. The info-gap concept responds to severe uncertainty that is unbounded or whose bound is unknown. It is not surprising that min-max and info-gap robustness analyses sometimes agree on their policy recommendations, and sometimes disagree, as has been discussed elsewhere.[(40)]
>
> Ben-Haim (2012, pp. 1643-1644)
>
> Note: citation (40) in Ben-Haim (2012), refers to Ben-Haim et al. (2009, pp. 1061-1062), which puts forth an even more egregiously flawed, hence a more seriously misleading, comparison of info-gap robustness and maximin robustness. A review of Ben-Haim et al. (2009) can be found elsewhere[2].

B A Maximin Theorem

All we need to do to prove formally and rigorously that info-gap decision theory's two core models are indeed maximin models, is to identify the two instances of the generic maximin model (13) that correspond to these core models. So consider the following two simple instances of (13) .

Generic maximin object in (13)	Instance I	Instance II
x	α	(q,α)
s	u	u
X	$[0,\infty)$	$Q\times[0,\infty)$
$S(x)$	$U(\alpha,\tilde{u})$	$U(\alpha,\tilde{u})$
$f(x,s)$	α	α
$con(x,s)$	$r_c\le r(q,u)$	$r_c\le r(q,u)$

Note: in *Instance I* the objects q and $\tilde{u}$ are fixed and given, and in *Instance II* the object $\tilde{u}$ is fixed and given.

Keep in mind the following:

Generic maximin model: $z^* := \max_{x\in X}\ \min_{s\in S(x)}\ \{f(x,s) : con(x,s), \forall s\in S(x)\}$ (13)

Info-gap robustness model: $\hat{\alpha}(q,r_c) := \max_{\alpha\ge 0}\ \{\alpha : r_c\le r(q,u), \forall u\in U(\alpha,\tilde{u})\}$ (5)

Info-gap robust-satisficing decision model: $\hat{\alpha}(r_c) := \max_{q\in Q,\alpha\ge 0}\ \{\alpha : r_c\le r(q,u), \forall u\in U(\alpha,\tilde{u})\}$ (6)

► THEOREM 1. *Info-gap's robustness model (5) and info-gap's robust-satisficing decision model (6) are simple maximin models. Indeed, they are simple instances of the generic maximin model defined by (13).*

[2] See http://info-gap.moshe-online.com/reviews/review_12.html

PROOF. Substituting the specification of *Instance I* in the maximin model (13) yields the following simple maximin model:

$$z^* := \max_{x\in X} \min_{s\in S(x)} \{f(x,s) : con(x,s), \forall s \in S(x)\} \tag{21}$$

$$= \max_{\alpha\geq 0} \min_{u\in U(\alpha,\tilde{u})} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\} \tag{22}$$

$$= \max_{\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}. \tag{23}$$

This is none other than info-gap's robustness model. And repeating the exercise with *Instance II*, yields the following simple maximin model:

$$z^* := \max_{x\in X} \min_{s\in S(x)} \{f(x,s) : con(x,s), \forall s \in S(x)\} \tag{24}$$

$$= \max_{q\in Q,\alpha\geq 0} \min_{u\in U(\alpha,\tilde{u})} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\} \tag{25}$$

$$= \max_{q\in Q,\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\} \tag{26}$$

which is none other than info-gap's robust-satisficing decision model. ◀

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