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Risk Analysis 101
Rhetoric in risk analysis, Part I: Wald's Mighty Maximin Paradigm*

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Preface

At the end of 2006 I posted on my website a short article entitled *Eureka! Info-Gap is Worst Case Analysis (Maximin) in Disguise!* where I set out a formal, rigorous proof that info-gap's robust-satisficing decision model is a (Wald) maximin model[1]. Since then I outlined similar formal proofs in other articles, including peer-reviewed articles, and I posted on my website a wealth of material supplementing this fact[2].

Over the years I repeatedly called the attention of many info-gap scholars, including Prof. Yakov Ben-Haim, the Father of info-gap decision theory, to the misleading rhetoric in the info-gap literature concerning the maximin connection. Regrettably, the misconceptions about this connection continue to be promulgated in the professional literature, including peer-reviewed journals such as *Risk Analysis,* whose referees should know better.

They should know better because this matter is as good as self-evident. Namely, it can be settled by inspection. For the question is this: is the model on the right hand-side an instance of the model on the left hand-side?

Prototype Maximin model	Info-gap's robust-satisficing decision model	
$\max\limits_{y\in Y}\ \min\limits_{s\in S(y)}\ \{f(y,s) : con(y,s), \forall s\in S(y)\}$	$\max\limits_{q\in Q,\alpha\geq 0}\ \{\alpha : r_c \leq r(q,u), \forall u\in U(\alpha,\tilde{u})\}$	(1)

where $con(y, s)$ denotes a list of constraints on the (y, s) pairs.

The rhetoric in the info-gap literature on this issue has it that the two models "are different". For, consider this:

> These two concepts of robustness—min-max and info-gap—are different, motivated by different information available to the analyst. The min-max concept responds to severe

*This article was written for the **Risk Analysis 101 Project** to provide a Second Opinion on pronouncements on the relationship between Wald's maximin paradigm and info-gap's robust-satisficing approach to decision making under severe uncertainty, published recently in *Risk Analysis*. See Risk-Analysis-101.moshe-online.com.

[1] See http://www.moshe-online.com/maximin/proof_a.pdf

[2] See http://info-gap.moshe-online.com

> uncertainty that nonetheless can be bounded. The info-gap concept responds to severe uncertainty that is unbounded or whose bound is unknown. It is not surprising that min-max and info-gap robustness analyses sometimes agree on their policy recommendations, and sometimes disagree, as has been discussed elsewhere.[(40)]
>
> Ben-Haim (2012, p. 7)

where reference [40] is Ben-Haim et al. (2009).

The implication therefore must be that, *Risk Analysis* referees are apparently of the opinion that the following model, where $\mathbb{R}$ denotes the real line, namely $\mathbb{R} := (-\infty, \infty)$, is not a minimax model:

$$z^* := \min_{x\in\mathbb{R}} \max_{y\in\mathbb{R}} \{x^2 + 2xy - y^2\}. \tag{2}$$

Or, could it be that these referees hold that, insofar as *Risk Analysis* is concerned, the interval $(-\infty, \infty)$ is bounded !!

One wonders ...

The incontestable fact obviously is that the above model is a perfectly kosher minimax model and the real line $\mathbb{R}$ remains unbounded.

The conclusion therefore must be that *Risk Analysis* referees are unaware of the fact that info-gap's robustness model and info-gap's robust-satisficing decision model are both maximin models. Specifically, they are unaware that these models are rather simple instances of the following prototype maximin model[3]:

$$z^\circ := \max_{y\in Y} \min_{s\in S(y)} \{f(y,s) : con(y,s), \forall s \in S(y)\}. \tag{3}$$

Or, if you will, these models are simple instances of the following "textbook" maximin model:

$$z' := \max_{y\in Y} \min_{s\in S(y)} g(y,s). \tag{4}$$

This being so, the implication therefore is that *Risk Analysis* referees second the absurd proposition that a simple *instance* of a *prototype* model is capable of representing situations that the prototype itself cannot represent. Namely, *Risk Analysis* referees accept the astounding proposition that while maximin models cannot handle unbounded uncertainty spaces, info-gap's robustness model indeed can!

Again, one wonders ...

It is important to take note that claims that info-gap's robust-satisficing decision model is not a maximin model are based on a comparison of these two models:

Maximin model	Info-gap's robust-satisficing decision model	
$\max_{q\in Q} \min_{u\in U(\alpha',\tilde{u})} r(q,u)$	$\max_{q\in Q,\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$	(5)

where α' is a given value of α.

But the point to note here is that this is a **non sequitur** *par excellence*. That is, the fact that the model on the left hand-side of (5) is dissimilar from the model on the right hand-side of (5) does not imply that the latter is not a maximin model.

Indeed, it is elementary to show that info-gap's robust-satisficing decision model is a maximin model. It is therefore mind boggling that info-gap scholars who base their claims on the comparison shown in (5), do not bother to consider the following comparison:

Maximin model	Robust-satisficing decision model	
$\max_{q\in Q,\alpha\geq 0} \min_{u\in U(\alpha,\tilde{u})} \{h(q,\alpha,u) : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$	$\max_{q\in Q,\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$	(6)

[3]Here $con(y,s)$ denotes a list of constraints on the (y,s) pairs.

Because, the fact that info-gap's robust-satisficing decision model is an instance of the maximin model shown in (6) simply stares one in the face!

So, again, one wonders ...

This state of affairs raises a number of questions. For instance, consider these two:

- Considering how easy it is to show/prove/verify that info-gap's robustness model and info-gap's robust-satisficing decision model are both maximin models, on what grounds do info-gap scholars claim, and *Risk Analysis* referees apparently concur, that these models are not maximin models?
- Why is it important to be clear on the fact that info-gap's robustness model and info-gap's robust-satisficing decision model are simple maximin models?

I take up the first question in the sequel. At this stage I address only the second question whose answer is in four parts:

- Info-gap decision theory is being proclaimed a new theory that is radically different from all current theories on decision under uncertainty. So, showing that its two core models are in fact simple instances of the most famous non-probabilistic robustness model used is the broad area of decision making, risk analysis etc., demonstrates how groundless this claim is. But more importantly, this fact raises serious questions about the narrative in the info-gap literature on Wald's maximin model, worst-case analysis, control theory, and so on. In short, this fact calls into question statements made in the info-gap decision theory about classic decision theory (Luce and Raiffa 1957, Resnik 1987, French 1988) and robust optimization (Gupta and Rosenhead 1968, Rosenhead et al. 1972, Mulvey et al. 1995, Bai et al. 1997, Kouvelis and Yu 1997, Ben-Tal and Nemirovski 1999, 2002, Bertsimas and Sim 2004, Ben-Tal et al. 2006, Ben-Tal et. al 2009, Bertsimas et al. 2011, Gabrel et al. 2012). It is important that readers of info-gap publications take this fact into account!
- The info-gap literature is saturated with misleading pronouncements on Wald's maximin paradigm and its many variant models: on its capabilities and limitations and its relation to info-gap's robustness model and info-gap's robust-satisficing decision model. It is regrettable that such pronouncements have found their way into peer-reviewed journals, such as *Risk Analysis*. It is important therefore to dispense with the fallacies about Wald's maximin paradigm that continue to be disseminated by peer-reviewed journals such as *Risk Analysis*.
- It is important that readers take special note of the following facts. Articles, such as Ben-Haim (2012), denying that info-gap's robust-satisficing decision model is a maximin model, and articles such as Schwartz et al. (2010), seeking to promote info-gap's robust-satisficing approach as a new *normative standard of rational decision making,* are engaged in a blatant misrepresentation of the state of the art in the broad area of *decision making* especially of the field of *robust optimization*.
- Indeed, in spite of the fact that both info-gap's robust-satisficing decision model and info-gap's robustness model are simple *robust optimization* models, not a single reference can be found to *robust optimization* in these two articles, nor in the three books on info-gap decision theory (Ben-Haim 2001, 2006, 2010). In fact, it would seem that every effort is made to avoid any discussion on *robust optimization,* and this in spite of the fact that the robust-satisficing approach advocated by info-gap decision theory is a simplistic, indeed, naive *robust optimization* approach.

It is important that referees of journals such as *Risk Analysis* be aware of these facts and their implications.

A close examination of info-gap's misleading rhetoric on the maximin connection reveals that info-gap scholars, and by implication *Risk Analysis* referees, have serious misconceptions about the following:

- The difference between *local* and *global* worst-case analysis.
- The difference between *local* and *global* robustness.
- The difference between robustness with respects to *payoffs* and robustness with respect to *constraints.*
- The relation between a *prototype* model and its *instances.*

These misconceptions are merely touched on in this article, for its main objective is to introduce referees of journals, such as *Risk Analysis,* to the rhetoric in the info-gap literature on the relationship between this theory and Wald's maximin paradigm.

The rhetoric in the info-gap literature surrounding the profound incongruity between the severity of the uncertainty postulated by info-gap decision theory, and the model of local robustness that the theory deploys for the management of this uncertainty, will be discussed in a separate article entitled *Rhetoric in risk analysis, Part II: Anatomy of a Peer-reviewed Voodoo Decision Theory.*

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1 Introduction

One can certainly argue intelligently about certain aspects of the relationship between Wald's maximin paradigm (Wald 1939, 1945, 1950; Luce and Raiffa 1957, Resnik 1987, French 1988) and info-gap decision theory (Ben-Haim 2001, 2006, 2010). For instance, it most certainly makes a lot of sense to ask whether info-gap's robustness model is a very simple maximin model or just a simple maximin model. Or, whether Wald's maximin decision rule is much more general and powerful than info-gap's robust-satisficing decision rule, or just more general and powerful. And, it most certainly makes sense to ask which maximin model is best suited for explaining why info-gap's robustness model and info-gap's robust-satisficing decision model are maximin models.

But no amount of rhetoric can change the following easily proved and demonstrated fact about the essence of this relationship:

Fact 1.1 *Both info-gap's robustness model and info-gap's robust-satisficing decision model are maximin models.*

Recall that info-gap decision theory (Ben-Haim 2001, 2006, 2010) is based on the following two core models. The first defines the robustness of decision $q \in Q$, the second determines the robustness of the most robust decision:

Info-gap's robustness model:

$$\hat{\alpha}(q, r_c) := \max_{\alpha \geq 0} \{\alpha : r_c \leq r(q, u), \forall u \in U(\alpha, \tilde{u})\} \ , \ q \in Q \tag{7}$$

Info-gap's robust-satisficing decision model:

$$\hat{\alpha}(r_c) := \max_{q \in Q, \alpha \geq 0} \{\alpha : r_c \leq r(q, u), \forall u \in U(\alpha, \tilde{u})\} \tag{8}$$

where

Q = set of available decisions.

$\mathscr{U}$ = set of all possible/plausible values of parameter u.

$\tilde{u}$ = point estimate of the true (unknown) value of the parameter of interest u.

$U(\alpha, \tilde{u})$ = neighborhood of size α around $\tilde{u}$.

$r(q, u)$ = performance level of decision q given u.

r_c = critical performance level.

Now, maximin models come in a variety of forms. Still, all maximin models have a basic characteristic in common. Namely, all are transliterations of the following "rule":

> The maximin rule tells us to rank alternatives by their worst possible outcomes: we are to adopt the alternative the worst outcome of which is superior to the worst outcome of the others.
>
> Rawls (1971, p. 152)

In other words, any transliteration of this Rule into a well-defined (not necessarily mathematical) model is a maximin model. And out of the numerous prototype maximin models that are relevant to this discussion, we shall focus on the following:

Maximin model:

$$z^* := \max_{y \in Y} \min_{s \in S(y)} \{f(y, s) : con(y, s), \forall s \in S(y)\} \tag{9}$$

where

Y = set of available *alternatives*.

$\mathbb{S}$ = set of all possible/plausible of state s.

$S(y)$ = set of *states* associated with alternative y.

$f(y,s)$ = *payoff* associated with the (y,s) pair.

$con(y,s)$ = list of *constraints* on the (y,s) pairs[4].

The characteristic that makes this model a maximin model is then that each alternative $y \in Y$ is assessed according to its *worst outcome* and the best alternative is that whose worst outcome is best. The "worst-case" orientation of this model is manifested in:

- The iconic expression $\min_{s\in S(y)}$ that represents worst-case analysis with respect to the *payoff* $f(y,s)$.
- The iconic expression $\forall s \in S(y)$ that represents a worst-case requirement with respect to the *constraints* $con(y,s)$.

I should point out that Fact 1.1 is an immediate implication of the following:

Fact 1.2 *Both info-gap's robustness model (7) and info-gap's robust-satisficing decision model (8) are simple, transparent instances of the prototype maximin model specified in (9).*

For the purposes of this discussion it suffices to point out that an *instance* of a *prototype* model is a model obtained by *instantiating,* namely specifying, one or more of the unspecified parameters of the prototype model. Therefore, the instances of a prototype model belong to the same class/family of models that the prototype represents. Think about a prototype model as a *set* containing models that share certain common properties. Each member of this set is an *instance* of the prototype.

Thus, the expression $3x+b$ is an instance of the expression $ax+b$, where a and b are unspecified parameters. Similarly, an instance of a polynomial is a polynomial; an instance of the quadratic equation $ax^2+bx+c=0$ where $a>0$, b and c are parameters, is a quadratic equation; an instance of a linear programming model is a linear programming model, and so on[5].

In particular, an instance of the prototype maximin model (9) is a model that is obtained by specifying one or more of the objects/constructs comprising this model. These objects/constructs are: the set Y, the sets $S(y), y \in Y$, the objective function $f = f(y,s)$ and the list of constraints $con(y,s)$. For example, the instance specified by $Y \leftarrow \mathbb{R}$, $S(y) \leftarrow \mathbb{R}, \forall y \in Y$, and by an empty list of constraints $(con(y,s))$, yields the following maximin model:

$$\max_{y\in\mathbb{R}} \min_{s\in\mathbb{R}} f(y,s). \tag{10}$$

Now, the argument that info-gap scholars are in the habit of making to purport that info-gap's robust-satisficing decision model is not a maximin model maintains the obvious. Namely, that info-gap's robust-satisficing decision model (8) is *different* from the following maximin model:

$$\max_{q\in Q} \min_{u\in U(\alpha,\tilde{u})} r(q,u) \ , \ \text{(for some given value of } \alpha). \tag{11}$$

In other words, the argument goes like this: given that (11) is a maximin model and that (8) is so different from (11), how can (8) possibly be a maximin model?!

But, to see that this argument proves nothing, observe that although the model in (11) is quite different from the model in (9), both models are perfectly kosher maximin models, and so is (2).

[4]For convenience, assume that in the definition/construction of set Y, there are constraints only on y, and in that of the sets $S(y), y \in Y$, there are constraints only on s (for a given value of y).

[5]The values assigned to the parameters that specify an instance of a model must, of course, comply with restrictions imposed on these parameters by the prototype (parent) model.

By analogy, the fact that $p(x) := a + bx + cx^2 + dx^3$ is a polynomial and that $q(x) := 1 - x^2$ is different from $p(x)$ does not imply that $q(x)$ is not a polynomial. Nor does it imply that q is not an instance of p.

So it is important to remind info-gap scholars and *Risk Analysis* referees of the following fact of life:

Fact 1.3 *The fact that a model, say Model A, is "different" from a given maximin model, e.g. the model defined in (11), does not imply that Model A is not a maximin model. Nor does it imply that Model A is not an instance of the given maximin model.*

The bottom line is then this:

Fact 1.4 *Any instance of a prototype maximin model is a maximin model. Thus, any instance of (9) is a maximin model.*

No amount of rhetoric can change these basic facts.

2 A simple Theorem

Consider the two instances of the prototype maximin model (9) whose specifications are shown in Figure 1. Note that the only difference between these two instances is in the specification of y and Y.

Instance I		**Instance II**	
$y \leftarrow \alpha$	(12)	$y \leftarrow (q, \alpha)$	(19)
$Y \leftarrow [0, \infty)$	(13)	$Y \leftarrow Q \times [0, \infty)$	(20)
$s \leftarrow u$	(14)	$s \leftarrow u$	(21)
$\mathbb{S} \leftarrow \mathscr{U}$	(15)	$\mathbb{S} \leftarrow \mathscr{U}$	(22)
$S(y) \leftarrow U(\alpha, \tilde{u})$	(16)	$S(y) \leftarrow U(\alpha, \tilde{u})$	(23)
$f(y, s) \leftarrow \alpha$	(17)	$f(y, s) \leftarrow \alpha$	(24)
$con(y, s) \leftarrow r_c \leq r(q, u)$	(18)	$con(y, s) \leftarrow r_c \leq r(q, u)$	(25)

Figure 1: Specifications of two instances of the maximin model defined in (9)

Theorem 2.1 *Both info-gap's robustness model (7) and info-gap's robust-satisficing decision model (8) are maximin models. More specifically, info-gap's robustness model is the instance of (9) specified by Instance I, and info-gap's robust-satisficing model is the instance of (9) specified by Instance II.*

Proof. All we have to do is to instantiate the maximin model given in (9) according to the two specifications. For Instance I we obtain

$$z^* := \max_{y \in Y} \min_{s \in S(y)} \{f(y, s) : con(y, s), \forall s \in S(y)\} \tag{26}$$

$$= \max_{\alpha \geq 0} \min_{u \in U(\alpha, \tilde{u})} \{\alpha : r_c \leq r(q, u), \forall u \in U(\alpha, \tilde{u})\} \tag{27}$$

$$= \max_{\alpha \geq 0} \{\alpha : r_c \leq r(q, u), \forall u \in U(\alpha, \tilde{u})\}. \tag{28}$$

Hence, info-gap's robustness model (7) is an instance of the maximin model defined by (9). And for Instance II we obtain

$$
\begin{aligned}
z^* :&= \max_{y\in Y} \min_{s\in S(y)} \{f(y,s) : con(y,s), \forall s \in S(y)\} && (29)\\
&= \max_{q\in Q,\alpha\geq 0} \min_{u\in U(\alpha,\tilde{u})} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\} && (30)\\
&= \max_{q\in Q,\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\} && (31)
\end{aligned}
$$

Hence, info-gap's robust-satisficing decision model (8) is also an instance of the maximin model defined by (9). *QED*

No amount of rhetoric can change this fact.

3 Maximin games

It is convenient and instructive to think about maximin models as two-players *games* featuring the decision maker (DM), who controls the value of the *alternative* ($y \in Y$) and *Nature*, who controls the value of the *state* ($s \in \mathbb{S}$) variable. It is assumed that the DM plays first and that her objective is to obtain the best possible *outcome. Nature* plays second, knowing what value of y was selected by the DM. *Nature's* objective is to make sure that the DM attains the *worst* possible outcome associated with the alternative chosen by the DM.

Symbolically, such games can be formulated as follows:

$$
v := \underset{y\in Y}{\text{best}} \ \underset{s\in S(y)}{\text{worst}} \ O(y,s) \tag{32}
$$

where $S(y)$ denotes the set of states associated with alternative y and $O(y,s)$ represents the *outcome* generated by the pair (y,s). The $\underset{y\in Y}{\text{best}}$ and $\underset{s\in S(t)}{\text{worst}}$ operations are based on a suitable *preference* order over the set of possible/plausible outcomes $\{O(y,s) : y \in Y, s \in S(y)\}$.

In the classic case, where the outcomes are real-numbers ($O(y,s) = f(y,s)$) representing *payoffs*, and the preference relation is the conventional "the larger the better" rule, this maximin model yields the following "textbook" prototype maximin model:

$$
v := \max_{y\in Y} \min_{s\in S(y)} f(y,s). \tag{33}
$$

The iconic expression $\min_{s\in S(y)}$ represents worst-case robustness with respect to the payoff $f(y,s)$.

Now, suppose that the alternatives $y \in Y$ are required to satisfy *constraints* that depend on the state variable $s \in \mathbb{S}$. Let then

$$
con(y,s) = \text{ list of constraints on the } (y,s) \in Y \times \mathbb{S} \text{ pairs.} \tag{34}
$$

An *outcome* in this case must therefore represent how "good" the pair (y,s) is with respect to the payoff $f(y,s)$ it generates and whether or not this pair satisfies the constraints $con(y,s)$. So, let such an outcome be represented by a pair $O(y,s) = (c,p)$ where p represents the payoff $f(y,s)$ and c represents the status of the constraints $con(y,s)$:

$$
c = \begin{cases} + & , \quad \text{the constraints } con(y,s) \text{ are satisfied} \\ - & , \quad \text{the constraints } con(y,s) \text{ are violated} \end{cases} \tag{35}
$$

For instance, the outcome $O(y,s') = (+,45)$ indicates that the pair (y,s') satisfies the constraints and generates a payoff of \$45, whereas the outcome $O(y,s'') = (-,90)$ indicates that the pair (y, s'') violates the constraints, but would have otherwise generated a payoff of \$90 (what a pity!).

As is invariably the case in *constrained* optimization, *lexicographic priority* is given to *constraint satisfaction:* an *optimal* solution must be a *feasible* solution, hence infeasible solutions, that is solutions that violate the constraints, are discarded at the outset.

So from the decision maker's point of view, violation of the constraints $con(y, s)$ should be avoided. Hence, for example, the decision maker would prefer the outcome $(+, -3000)$ to the outcome $(-, 900000)$, even though she would greatly prefer a payoff of \$900000 to a payoff (loss) of $-\$3000$. More generally, the decision maker would prefer an outcome $(+, A)$ to an outcome $(-, B)$ regardless of the values of the payoffs A and B.

This means that, with no loss of generality, the decision maker can ignore any alternative $y \in Y$ such that $O(y, s) = (-, C)$ for some $s \in S(y)$. Differently put, the decision maker can restrict the choice of $y \in Y$ to alternatives that satisfy the constraints for $\boxed{\text{all}}$ the values of s associated with them. In this case, the preference among these alternatives can be based on the good old "larger is better" preference order over the payoffs $f(y, s), s \in S(y)$.

The *lexicographic preference order* outlined above can be captured by a modified payoff function, $F = F(y, s)$, defined as follows:

$$F(y,s) := \begin{cases} f(y,s) & \text{, pair } (y,s) \text{ satisfies the constraints } con(y,s) \\ -\infty & \text{, pair } (y,s) \text{ violates the constraints } con(y,s) \end{cases} \text{ , } y \in Y, s \in S(y). \quad (36)$$

This yields the prototype maximin model

$$v = \max_{y \in Y} \min_{s \in S(y)} F(y, s). \quad (37)$$

The large penalty $(-\infty)$ for violating the constraints $con(y, s)$ deters the decision maker from selecting any alternative $y \in Y$ that violates the worst-case robustness constraint $con(y, s), \forall s \in S(y)$. Hence, this "textbook" prototype maximin model is equivalent to a maximin model where the outcomes are specified by $f(y, s)$ and the (y, s) pairs are subject to the worst-case robustness constraint $con(y, s), \forall s \in S(y)$.

In short, incorporating the constraints $con(y, s)$ in the maximin game yields the following maximin representation of the game:

$$v := \max_{y \in Y} \min_{s \in S(y)} \{f(y, s) : con(y, s), \forall s \in S(y)\}. \quad (38)$$

Last but not least, in simpler cases where the payoffs $f(y, s), y \in Y, s \in S(y)$ do not depend on s, namely in cases where $f(y, s) = h(y)$ for some real-valued function h, the prototype maximin model of the game is as follows:

$$v := \max_{y \in Y} \{h(y) : con(y, s), \forall s \in S(y)\}. \quad (39)$$

The absence of the iconic expression $\min\limits_{s \in S(y)}$ from this formulation, gives notice that this prototype maximin model does not seek worst-case robustness with respect to the payoffs because the payoff $f(y, s) = h(y)$ is independent of the state variable s.

Note that in the context of such models the outcome associated with a (y, s) pair is equal to either $(+, h(y))$ or $(-, h(y))$, depending on whether the pair satisfies the constraints $con(y, s)$. Therefore, the existence of a worst outcome (for each alternative y) is a non-issue: **there always is** at least one worst outcome:

- If the constraints $con(y, s)$ are satisfied for all s in $S(y)$, then all the elements of $S(y)$ are worst cases of u and the worst outcome is equal to $(+, h(y))$.
- If the constraints $con(y, s)$ are not satisfied for all s in $S(y)$, then all the elements of $S(y)$ that violate these constraints are worst cases of u and the worst outcome is equal to $(-, h(y))$.

In short: in the case of maximin models of the type specified in (39), the worst outcome associated with alternative y is equal to either $(+, h(y))$ or $(-, h(y))$. Meaning that one is a priori assured of the existence of a worst case.

Thus, to turn the tables on the misleading info-gap rhetoric, exemplified for instance in the statement quoted above from Ben-Haim (2012), it is important to be clear on the facts of the matter.

The info-gap rhetoric has it that in contrast to a maximin analysis, where the existence of a worst case is an issue, in info-gap decision theory a worst outcome does not enter into the analysis and therefore the uncertainty space can be unbounded. Hence, an info-gap analysis is not a worst case analysis, hence it is not a Maximin analysis.

The fact of the matter is of course that the existence of worst outcomes is never an issue in the case of maximin models of the type specified by (39) because one is assured that a worst case exists as a matter of principle in the framework of such models. Hence, given that info-gap's robustness model and info-gap's robust-satisficing model are instances of (39), it follows that the existence of worst outcomes is not an issue in info-gap robustness games either. But, this is so for reasons that are totally different from those given by the info-gap rhetoric.

Info-gap robustness game

Let us now examine how the maximin game is manifested in the context of info-gap's robustness model, namely the model:

$$\hat{\alpha}(q, r_c) := \max_{\alpha \geq 0} \{\alpha : r_c \leq r(q, u), \forall u \in U(\alpha, \tilde{u})\} \ , \ q \in Q. \tag{40}$$

Note that here q is fixed and given, and so are $\tilde{u}$ and r_c.

In this model the decision maker controls the value of α and *Nature* controls the value of u, hence α represents an "alternative", and u represents a "state". For a given choice of α by the decision maker, *Nature* selects the worst u in $U(\alpha, \tilde{u})$ with respect to the constraint $r_c \leq r(q, u)$.

The worst u in $U(\alpha, \tilde{u})$ is determined as follows:

- If there is a u in $U(\alpha, \tilde{u})$ such that $r_c > r(q, u)$, then any such u is a worst case. The worst outcome is $(-, \alpha)$, observing that the $-$ indicates that the performance constraint $r_c \leq r(q, u)$ is violated somewhere on $U(\alpha, \tilde{u})$.
- If $r_c \leq r(q, u), \forall u \in U(\alpha, \tilde{u})$, then any u in $U(\alpha, \tilde{u})$ is a worst case (and a best case), and the worst outcome is $(+, \alpha)$.

The constraint $r_c \leq r(q, u)$ in the info-gap's robustness model (40) discards all the inadmissible values of α, namely values of α for which the outcome is $(-, \alpha)$.

This is illustrated graphically in Figure 2, where the uncertainty space is represented by the large rectangle, the neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$ are represented by circles of radius α around $\tilde{u}$, and the set of acceptable values of u is represented by the shaded area.

The worst values of u associated with the choice of alternative $\alpha = \alpha'$ are represented by the white area in the neighborhood $U(\alpha', \tilde{u})$. The set of all the worst values of u associated with the choice of alternative $\alpha = \alpha''$ comprises the entire neighborhood $U(\alpha'', \tilde{u})$.

The info-gap robustness of decision q, denoted $\hat{\alpha}(q, r_c)$, is equal to the radius of the largest circle contained in the shaded area. This is equal to the best (with respect to alternative α) worst (with respect to $u \in U(\alpha, \tilde{u})$) payoff ($\alpha$) subject to the local robustness constraint $r_c \leq r(q, u), \forall u \in U(\alpha, \tilde{u})$.

4 Maximin formulations for all occasions

As I pointed out already, maximin models come in a variety of forms. This means that the maximin paradigm puts at our disposal an extremely pliable tool for the purpose of conducting worst-case robustness analyses. For, recall Rawls' (1971) formulation of the Maximin Rule stated above, and the following slightly modified versions thereof:

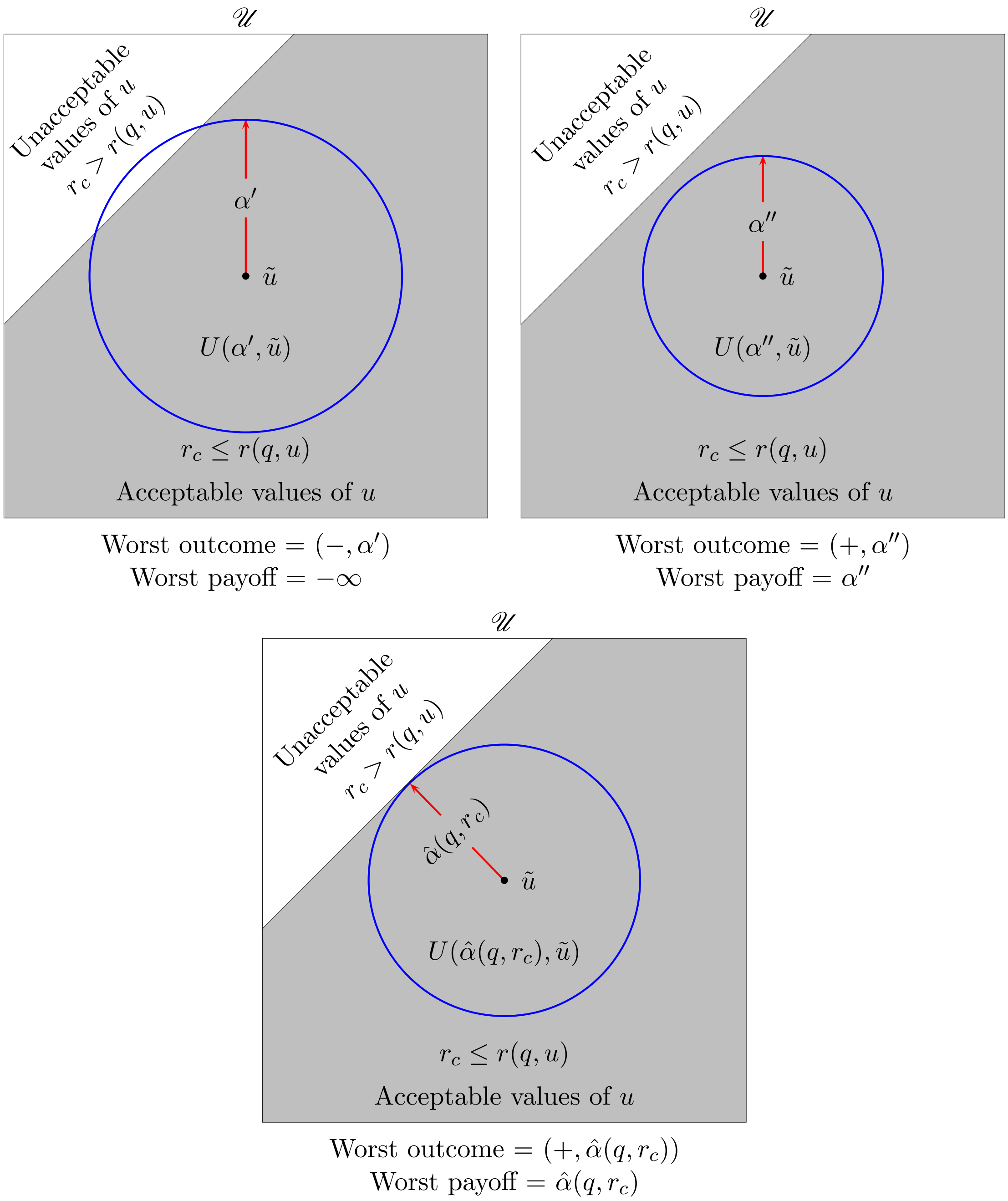


Figure 2: Info-gap's robustness game: $\hat{\alpha}(q,r_c) := \max\limits_{\alpha \geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$

- **Maximin Rule:**
  Rank alternatives by their worst outcomes, hence select the alternative the worst outcome of which is at least as good as the worst outcome of the others.

The question is then how do we put this idea to work? This is precisely where the crucial issue of modeling the Maximin Rule comes into play. Modeling the Maximin Rule implies giving the key concepts "alternative", "outcome", "worst", and "best" determinate meaning, giving them precise definitions and structure. And to illustrate this important point, we distinguish between the following three prototype maximin models[6]:

Full Monty maximin model:

$$\max_{y\in Y} \min_{s\in S(y)} \{f(y,s) : con(y,s), \forall s \in S(y)\} \tag{41}$$

Classic maximin model:

$$\max_{y\in Y} \min_{s\in S(y)} \ f(y,s) \tag{42}$$

MP maximin model:

$$\max_{y\in Y} \ \{f(y) : con(y,s), \forall s \in S(y)\} \tag{43}$$

As explained above, the distinction between the three models is due to the object(s) driving the robustness analysis, the objects being the *objective function*, f, and the *constraints*, *con*:

- The Full Monty model seeks robustness with respect to both the objective function and the constraints.
- The Classic model seeks robustness only with respect to the objective function.
- The MP model seeks robustness only with respect to the constraints.

Obviously, the Classic and MP models are simple instances of the Full Monty model. What is not so obvious apparently, is that these three models are in fact *equivalent* in the sense that each model yields the other two as instances. For instance, to show that the seemingly rather anemic MP model is actually as versatile as the impressive Full Monty model, consider the following MP maximin model:

$$\max_{x\in X, v\in\mathbb{R}} \ \{v : v \leq f(x,s), con(x,s), \forall s \in S(x)\}. \tag{44}$$

Also, note that[7]

$$\min_{z\in Z} h(z) \equiv \max_{v\in\mathbb{R}} \ \{v : v \leq h(z), \forall z \in Z\}. \tag{45}$$

We thus have,

$$\max_{x\in X, v\in\mathbb{R}} \ \{v : v \leq f(x,s), con(x,s), \forall s \in S(x)\}$$

$$\equiv \max_{x\in X} \max_{v\in\mathbb{R}} \ \{v : v \leq f(x,s), con(x,s), \forall s \in S(x)\} \tag{46}$$

$$\equiv \max_{x\in X} \min_{s\in S(x)} \ \{f(x,s) : con(x,s), \forall s \in S(x)\}. \tag{47}$$

This means that instances of the prototype Full Monty maximin model can be formulated as MP maximin models, and vice versa.

All this goes to show that analysts have at their disposal a wide range of maximin models to chose from. Hence, when it comes to formulating a given maximin problem, analysts can

[6]MP ≡ Mathematical Programming

[7]This modeling trick is used extensively in game theory, operations research and robust optimization (e.g. Ecker and Kupferschmid 1988, pp. 24-25; Kouvelis and Yu, 1997, p. 27).

use any one of these three prototype maximin models, depending on the problem they seek to solve and on their objectives.

Now, in view of what we have seen so far, it is clear that the most obvious way to relate info-gap's robust-satisficing decision model and info-gap's robustness model to the maximin model is through the prototype MP maximin model. Indeed, a simple "visual" comparison immediately shows that info-gap's robust-satisficing decision model and info-gap's robustness model are simple instances of the prototype MP maximin model. For instance, consider this:

MP maximin model	Info-gap's robust-satisficing decision model
$\max_{y\in Y} \{f(y) : con(y,s), \forall s \in S(y)\}$	$\max_{q\in Q,\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$

(48)

You don't have to be a risk analyst to figure out, indeed this is obvious by inspection, that the model on the right hand-side is an instance of the model on the left hand-side.

And yet, info-gap scholars maintain that info-gap's robust-satisficing decision model and info-gap's robustness model are not maximin models.

The question of course is: how is this possible?

An examination of the info-gap literature reveals that, **for reasons never made clear, indeed never given the slightest justification anywhere in this literature,** the claims denying that info-gap's robust-satisficing decision model and info-gap's robustness model are maximin models are based on comparisons such as this:

Maximin model	Info-gap's robust-satisficing decision model
$\max_{q\in Q} \min_{u\in U(\alpha',\tilde{u})} r(q,u)$	$\max_{q\in Q,\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$

(49)

where α' is a given value of α.

The argument is that the fact that these two models are dissimilar proves that info-gap's robust-satisficing decision model and info-gap's robustness model are not maximin models.

But, as I noted above, this is a **non sequitur** *par excellence*. The fact that in (49) the model on the left hand-side and the model on the right hand-side have different formulations and yield different results **does not** imply that the model on the right hand-side is not a maximin model. All that this comparison indicates is that ... the two models which are different (i.e. are different instances of the prototype (9)) typically yield different results. The important point is that, a quick look at (48) suffices to see at a glance that info-gap's robust-satisficing decision model and info-gap's robustness model are indeed maximin models.

Why info-gap scholars base their claims concerning the info-gap maximin connection on the comparison shown in (49) and not say, on a comparison shown in (48) is anybody's guess.

My main point is that arguing along these lines is totally equivalent to seeking to settle whether "function p defined by $p(x) = 1 + x^2 - x^6$ is a polynomial" by comparing it to some degree 5 polynomial, say $P(x) = 1 + ax + bx^2 - cx^5$, and concluding that because p is different from P, then p is not a polynomial!

Go figure!

5 The Instance that Roared

One of the absurdities advanced in the info-gap literature (e.g. Ben-Haim 2012, p. 7), in an effort to disassociate info-gap's robust-satisficing decision model from the maximin model, is that info-gap's robust-satisficing decision model (8) can handle a severe uncertainty that is beyond the capabilities of maximin models. To bring out the full dimensions of this absurd, which apparently is accepted by *Risk Analysis* referees, let us go back to Figure 1. The idea is to show that the two instances shown in Figure 1 are immeasurably less powerful than the prototype maximin model (9) itself.

Of particular interest to us is of course Instance II, namely the instance yielding info-gap's robust-satisficing decision model (8). My objective is to show that this instance is no more than a "pale shadow" of the mighty maximin model (9).

Consider first the specification $S(y) \leftarrow U(\alpha, \tilde{u})$, recalling that $y \leftarrow \alpha$ and that in the framework of info-gap decision theory

$$\begin{aligned} U(\alpha, \tilde{u}) :&= \text{ neighborhood of size } \alpha \text{ around } \tilde{u} && (50) \\ &= \text{set of all } u \in \mathscr{U} \text{ that are within a distance } \alpha \text{ from } \tilde{u}, && (51) \end{aligned}$$

where distances from $\tilde{u}$ are defined by some suitable measure (norm/metric) of distance on the uncertainty space $\mathscr{U}$.

In contrast, in the framework of the maximin model (9),

$$S(y) := \text{ set of states associated with alternative } y. \quad (52)$$

This means that whereas the maximin model (9) allows $S(y)$ to be *any subset* of the state space under consideration, the specification $S(y) \leftarrow U(\alpha, \tilde{u})$ restricts $S(y)$ to be a *neighborhood* of size α around $\tilde{u}$.

As far as mathematical modeling is concerned, there is a huge difference, both methodologically and practically, between the following two requirements that a set $V \subseteq \mathscr{U}$ must satisfy:

Requirement 1: V must be a $\boxed{\text{subset}}$ of $\mathscr{U}$. (53)

Requirement 2: V must be a $\boxed{\text{neighborhood}}$ in $\mathscr{U}$ around a given point in $\mathscr{U}$. (54)

Suffice it to say that the first is much less restrictive than the second: a *neighborhood* $U(\alpha, \tilde{u}) \subseteq \mathscr{U}$ is a *very special subset* of $\mathscr{U}$.

Methodologically, this means that the worst-case analysis conducted by Instance II in Figure 1 is inherently *local* in nature: it is conducted over *neighborhoods* around $\tilde{u}$. Consequently, the robustness model induced by the specification $S(y) \leftarrow U(\alpha, \tilde{u})$ is a model of *local* robustness.

Next, consider the specification $f(y, s) = \alpha$, recalling that according to Instance II, $y = (q, \alpha)$. The most significant implication of this specification is that the *objective function* of the instance induced by this specification, namely function $f = f(y, s)$, is independent of the state variable s. This means in turn that the model induced by this specification **does not seek robustness with respect to the objective function f.**

In other words, info-gap's robust-satisficing decision model seeks robustness **only** with respect the constraint $r_c \leq r(q, u)$: for a given choice of alternative $y = (q, \alpha)$, a *local* worst-case analysis is conducted on the neighborhood $U(\alpha, \tilde{u})$ to check whether the constraint $r_c \leq r(q, u)$ is satisfied for all $u \in U(\alpha, \tilde{u})$. If it is, then the choice $y = (q, \alpha)$ is admissible. If this worst-case requirement is not satisfied, then $y = (q, \alpha)$ is inadmissible.

The important point to note then is that the above specifications narrow down the all-embracing prototype maximin model (9) to a highly specialized model, namely a model that seeks *local* robustness **only** with respect to the *constraint* $r_c \leq r(q, u)$ **confining the search to the locale of $\tilde{u}$.** The implications of this observation are summarized in Figure 3.

The facts are clear as daylight. There is no comparison between the capabilities of the prototype maximin model defined in (9) to model and search for robustness, and its instance stipulated by Instance II in Figure 1, namely info-gap decision theory's robust-satisficing decision model (8).

And yet, statements such as that quoted above (Ben-Haim 2012, p. 7), which abound in info-gap publications, absurdly claim that this instance has the capabilities to perform feats that the powerful prototype that it derives from does not namely, handling **unbounded** uncertainty spaces. And what is so farcical in all this is that this instance proposes to deal with unbounded uncertainty spaces by 'going local', namely by in fact *ignoring* the severity of the uncertainty manifested in unbounded uncertainty spaces, and focusing instead on the neighborhood of a

Prototype	Instance specified by Instance II
$\max_{y\in Y} \min_{s\in S(y)} \{f(y,s) : con(y,s), \forall s \in S(y)\}$	$\max_{q\in Q,\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$

Type of robustness capable of seeking

	payoffs	constraints
local	$\surd$	$\surd$
global	$\surd$	$\surd$
other	$\surd$	$\surd$

	payoffs	constraints
local		$\surd$
global		
other		

Figure 3: A prototype and one of its many instances

point estimate $\tilde{u}$ that can be just a wild guess of the true (unknown) value of the parameter. Surely, the only way to put across the absurd in the rhetoric accompanying this instance is to refer to it as the *Instance that Roared.*

I call models of this type *voodoo* decision models and theories based on such models *voodoo decision theories.* In this context, the term *voodoo* has the same role it has in "voodoo economics", "voodoo science", "voodoo statistics", "voodoo mathematics", and so on (see Sniedovich 2010, 2012, 2012a, 2012b). The point of the sobriquet *voodoo* is to bring into sharp focus that the robust-satisficing decision models (8) in fact ignores completely the performance of decisions outside a given (bounded) neighborhood of the unbounded uncertainty space. I call the region of the uncertainty space that is outside this neighborhood the *No Man's Land* (see Sniedovich 2010, 2012, 2012a, 2012b). In the case of info-gap decision theory, the *No Man's Land* of decision q is the set

$$NML(q) := \mathscr{U} \setminus U(\alpha^*,\tilde{u}) \ , \ \alpha^* = \hat{\alpha}(q,r_c) + \varepsilon \tag{55}$$

where ε can be arbitrarily small (but positive).

This is illustrated in Figure 4 where the large rectangle represents the uncertainty space $\mathscr{U}$ and the small white circle represents the set $U(\alpha^*,\tilde{u})$. The info-gap robustness of decision q takes no account whatsoever of the performance of decision q over the black area representing the *No Man's Land* of decision q.

Note that, according to Ben-Haim (2006, p. 210), "... Most of the commonly encountered info-gap models are unbounded ...". This means that the size of set $U(\alpha^*,\tilde{u})$ in this figure is grossly exaggerated: it should be infinitesimally small. In other words, in most of the commonly encountered applications of info-gap decision theory, set $U(\alpha^*,\tilde{u})$ is minute (infinitesimally small) compared to its *No Man's Land.*

Thus, Figure 5 provides a much better representation of the inherently *local* orientation of info-gap's robustness model.

If this is not *voodoo* decision-making in the face of severe uncertainty, what is?

6 Info-gap rhetoric in action

In this section I take another look at the rhetoric in Ben-Haim (2012) regarding the relationship between info-gap decision theory and Wald's maximin paradigm. Since this rhetoric does not refer to any specific maximin model, it is instructive to conduct this discussion with the following two models in mind:

(The Prototype) Maximin model	(The Instance that Roared) Info-gap's robust-satisficing decision model
$\max_{y\in Y} \min_{s\in S(y)} \{f(y,s) : con(y,s), \forall s \in S(y)\}$	$\max_{q\in Q,\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$

(56)

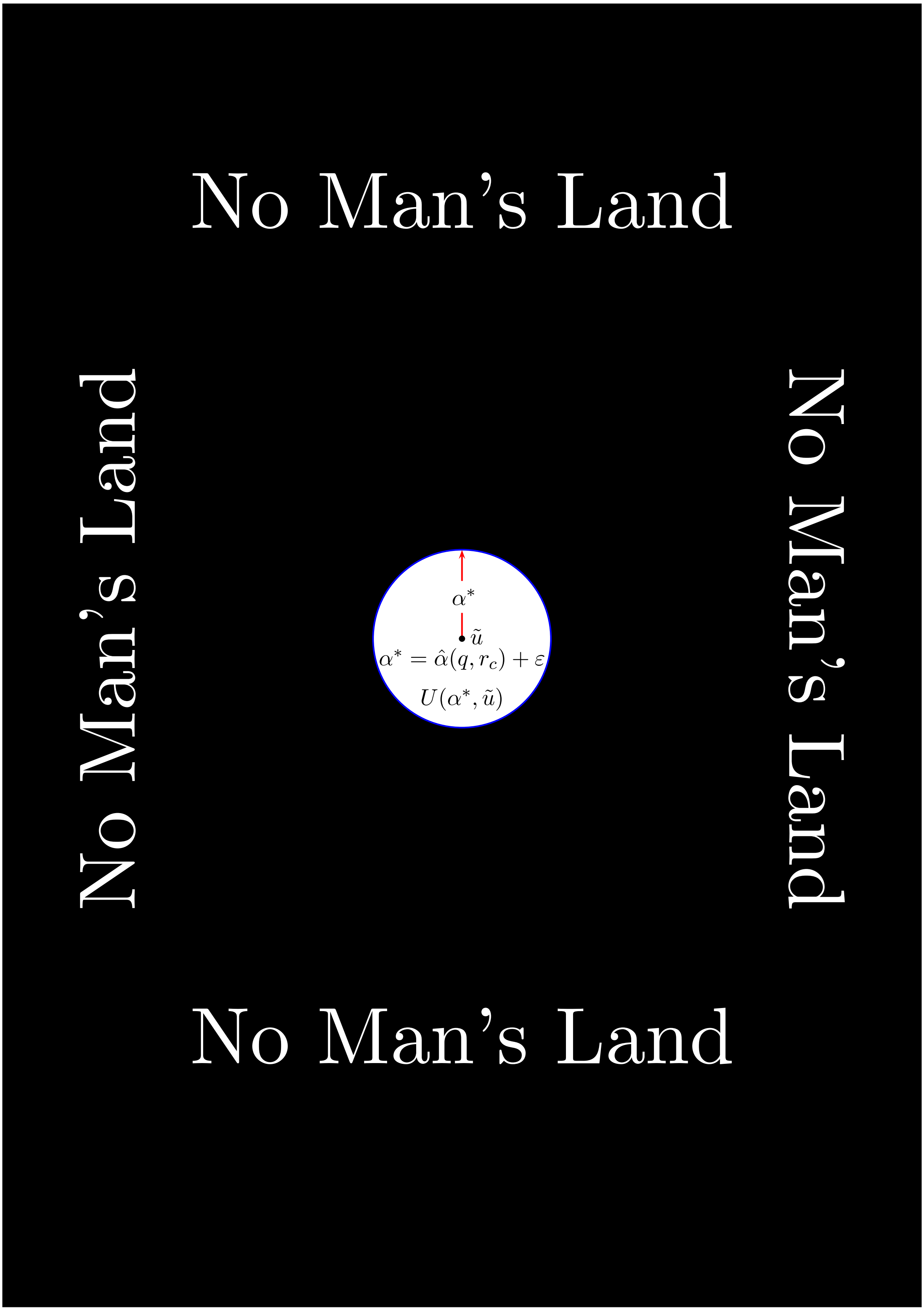


Figure 4: Info-gap's *No Man's Land* of decision q

Recall that the specification (instantiation) that yields the *Instance that Roared* is as follows (see Figure 1):

$$y \leftarrow (q,\alpha);\ s \leftarrow u;\ Y \leftarrow Q \times [0,\infty);\ S(y) \leftarrow U(\alpha,\tilde{u});\ f(y,s) \leftarrow \alpha;\ con(y,s) \leftarrow r_c \leq r(q,u) \tag{57}$$

On the agenda here is the text in Ben-Haim (2012, pp. 6-7) which advices on the similarities and differences between info-gap's measure of robustness and the measure of robustness that characterizes Wald's maximin (or rather min-max) paradigm.

The misleading message implied by this text is this:

- Unlike the min-max paradigm—which cannot cope with situations where the uncertainty space is unbounded, or unknown—info-gap's robust-satisficing decision model is capable of dealing with such difficult situations.
- In fact, info-gap decision theory was developed specifically to handle such situations.
- Info-gap's robust-satisficing decision model is not based on a worst-case analysis.
- Clearly then, info-gap's robust-satisficing decision model is definitely not a maximin model.

Let us then take a closer look at these claims.

6.1 What is a min-max strategy?

Consider this:

> A widely occurring operational distinction between risk analyses hinges on whether or not meaningful worst cases can be identified. When one can plausibly specify the worst events that can occur (and presuming we don't know probability distributions), then one might justifiably try to ameliorate these worst contingencies. This can be done in many different ways, and we will refer to this type of strategy as min-max analysis: minimizing the maximum damage.
>
> Ben-Haim (2012, p. 6)

To see how packed with errors and misconceptions this statement is, recall Rawls' formulation of Wald's min-max (maximin) paradigm:

> The maximin rule tells us to rank alternatives by their worst possible outcomes: we are to adopt the alternative the worst outcome of which is superior to the worst outcome of the others.
>
> Rawls (1971, p. 152)

The point is that from the standpoint of decision theory and optimization theory, an "outcome" refers to two things:

- The *payoff,* namely the value attained by the *objective function.*
- An indication whether the relevant *constraints* are satisfied or violated.

However, according to this quoted statement, "min-max analysis" engages only in "minimizing the maximum damage", the implication being that it is only an *objective function* analysis. The conclusion to be drawn from this paragraph is then that min-max models cannot perform worst-case analysis of *constraints.*

In fact, the phrase "minimizing the maximum damage" in Ben-Haim (2012) is doubly misleading. It is misleading not only because it, deliberately or inadvertently, gives the wrong impression that maximin models cannot seek robustness with respect to *constraints.* It is misleading on account of its attempt to thereby disassociate info-gap's robustness model from the so-called mini-max strategy which purportedly engages only in "minimizing the maximum damage".

It is also important to take note that a key implication of this statement is that info-gap's robust-satisficing decision model is not based on a worst-case analysis.

This, of course, is a blatant misrepresentation. The clause $\forall u \in U(\alpha, \tilde{u})$ in (8) is an iconic *local* worst-case clause: it requires the constraint $r_c \leq r(q, u)$ to be satisfied by the worst u in $U(\alpha, \tilde{u})$. In other words, it requires decision q to be robust, against deviations/perturbations in the value of $\tilde{u}$, in a worst-case sense.

6.2 Specification of meaningful worst contingencies

Next, consider this:

> The ability to implement a min-max analysis depends on identifying meaningful worst contingencies. This is feasible in many situations. The concept of a "meaningful worst case" depends on knowledge and judgment that may be within the risk analyst's competence. However, it is not usually sufficient to specify a worst case in some formal or abstract sense, such as the set of all contingencies that are consistent with the laws of science.
>
> Ben-Haim (2012, p. 6)

These comments apply not only to "a min-max analysis", they equally apply to info-gap modeling, namely to the specification of the uncertainty space $\mathscr{U}$ on which neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$ are defined.

That is, according to info-gap decision theory, the uncertainty space $\mathscr{U}$ contains all the neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$ and it represents the set of all the possible/plausible values of u. Thus, constructing the set $\mathscr{U}$ is completely equivalent to the construction of the state space $\mathbb{S}$ of a maximin model.

To wit, the specification of "meaningful worst contingencies" is an integral part of the specification of the state space of the maximin model, $\mathbb{S}$. And often the worst contingencies are not identified and specified at the outset, but rather are determined by the worst-case analysis. But the very same modeling issues that are encountered in specifying the state spaces of maximin models are encounter in specifying the uncertainty spaces of info-gap robustness models:

Maximin	Info-gap
$s :=$ state variable	$u :=$ uncertainty parameter
$\mathbb{S} :=$ set of all possible/plausible values of s.	$\mathscr{U} :=$ set of all possible/plausible values of u.

In both cases, worst-case contingencies are incorporated in the specification of the "set of possible/plausible contingencies", be it $\mathscr{U}$ or $\mathbb{S}$, either explicitly or otherwise.

Next, consider the following statement:

> Info-gap theory is not a worst case analysis. While there may be a worst case, one cannot know what it is and one should not base one's policy upon guesses of what it might be.
>
> Ben-Haim (2010, p. 9)

As I have already dealt with the fact that info-gap theory is indeed a worst case analysis *par excellence*, I need not respond to the blatant misrepresentation opening this statement. Rather, the point I want to single out for attention is that it is most interesting that Ben-Haim (2010) appears reluctant to estimate or "guess" the worst contingencies. Yet, Ben-Haim (2007, p. 2) is quite happy for info-gap's robustness analysis to be conducted around a point estimate $\tilde{u}$ that is no more than a wild guess of the true value of u:

> The best estimate, $\tilde{u}$, of an info-gap model of uncertainty is sometimes a wild guess, since in most cases the horizon of uncertainty, α, is unknown.
>
> Ben-Haim (2007, p. 2)

Proceeding then on the universally accepted maxim that *the results of an analysis are only as good as the estimates on which they are based,* doesn't it follow that the results generated by info-gap's robustness model can be no more than $\boxed{\text{wild guesses?}}$

6.3 The boundary between possible and impossible events

Next, consider this:

> Min-max analysis is most useful when the analyst is able to avoid vacuous specification of worst cases. However, when information is really scarce, for instance, when processes are poorly understood or changing, then even typical cases cannot be reliably identified. It may then be impossible to meaningfully specify the boundary between extreme but possible occurrences, and the impossible or negligible.
>
> Ben-Haim (2012, p. 6)

Again:

These comments apply not only to the min-max analysis but to info-gap modeling as well, namely to the specification of the uncertainty space $\mathscr{U}$ on which neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$ are defined.

That is, according to info-gap decision theory, the uncertainty space $\mathscr{U}$ contains $\boxed{\text{all}}$ the neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$ and represents the set of $\boxed{\text{all}}$ the possible/plausible values of u. Thus, constructing the set $\mathscr{U}$ is completely equivalent to the construction of the state space $\mathbb{S}$ of a maximin model.

So whatever difficulties are encountered in meaningfully specifying the boundary between extreme but possible occurrences, and the impossible or negligible, in the framework of a maximin model are also encountered in the framework of an info-gap robust-satisficing decision model.

In short, the whole issue of stipulating the boundaries between extreme but possible occurrences, and the impossible or negligible, is an integral part of specifying the uncertainty space of info-gap models just as it is an integral part of specifying the state space of maximin models:

Maximin	Info-gap
$s :=$ state variable	$u :=$ uncertainty parameter
$\mathbb{S} :=$ set of all possible/plausible values of s	$\mathscr{U} :=$ set of all possible/plausible values of u

Also, if the uncertainty that info-gap decision theory claims to deal with is so severe that it is impossible to meaningfully specify the boundary between extreme but possible occurrences, and the impossible or negligible, wouldn't it also be impossible to meaningfully specify the point estimate $\tilde{u}$?

And if the point estimate $\tilde{u}$ is allowed to be just a $\boxed{\text{wild guess,}}$ why shouldn't the worst cases also allowed to be wild guesses?

6.4 And here comes the cavalry

What happens in situations where the min-max strategy purportedly cannot cope with the severity of the uncertainty under consideration?

Simple: info-gap decision theory steps into the breach!

For consider the following paragraph:

> Nonetheless, even when worst cases cannot be meaningfully specified, the analyst still has data, understanding, and mathematical representations: models in the broad sense that we are using that term. It is simply that the analyst cannot responsibly specify the magnitude of error of these models. For instance, we have many models for long-range climate change, but the earnest scientific disputes over these models may preclude the ability to confidently bound the errors. Or, introducing a new species to an ecosystem, either deliberately as an genetically modified organism or inadvertently by invasion, may alter the ecosystem dynamics in unknown ways.
> In such situations one can still formulate and implement a robustness analysis. Info-gap theory has been developed precisely for the task. Lets discuss min-max and info-gap concepts of robustness.
>
> Ben-Haim (2012, p. 7)

That is, Ben-Haim (2012) and *Risk Analysis* referees would have us believe that maximin models cannot deal with the issues described in the first paragraph whereas info-gap's robust-satisficing model indeed can!

The absurd entailed by this paragraph is breathtaking. Because, the implication is that info-gap's robust-satisficing model (8), which as shown in Figure 1, is an instance of the maximin model (9), namely Instance II, has capabilities that the maximin model (9) itself does not possess!

The fact of the matter is of course that because the maximin model (9) is the prototype and info-gap's robust-satisficing model (8) is an instance thereof, if info-gap's robust-satisficing model (8) can indeed cope responsibly with the above difficulties, then it goes without saying that the maximin model (9) can. But more than that, as the prototype, this maximin model can cope with many other issues that info-gap's robust-satisficing model (8) cannot possibly cope with.

That said, the real question is: as a model of *local* robustness, how can info-gap's robustness model possibly handle responsibly situations where "the analyst cannot responsibly specify the magnitude of error of these models"?

6.5 Min-max question vs info-gap question

Next, consider this:

> The min-max concept of robustness responds to the question: How bad is the worst case? This is valuable information for the risk analyst and decision maker because if the worst case—after amelioration by a minmax analysis—is tolerable, then one can reasonably say that the system is robust to uncertainty.
>
> The info-gap concept of robustness responds to a different question: How wrong can the models be and still guarantee that the outcome is acceptable? This is useful for the risk analyst and decision maker because if the models can err enormously without preventing acceptable outcomes, then one can reasonably say that the system is robust to uncertainty.
>
> Ben-Haim (2012, p. 7)

There is a number of problematic issues in this paragraph that ought to be addressed. But, for our purposes it suffices to call attention to the following:

- As part of the ongoing effort to sharply differentiate between the maximin paradigm and info-gap decision theory, the above text misleadingly suggests that maximin models cannot address the question addressed by info-gap's robust-satisficing model.
- To be sure, one would expect that the formulation of the issue(s) addressed by a *prototype* would be different from the formulation of the issue(s) addressed by its various *instances,*

especially instances that are significantly simpler than the prototype. So, it is hardly surprising that the phrasing of the two questions in the above quote are different. However, this does not imply that the "info-gap robustness question" is not an instance of the "minimax robustness question".

- In fact, the two questions can be given a slightly different phrasing (from that given them in the quoted paragraph) so as to bring out that the "info-gap robustness question" is an instance of the "min-max robustness question".
- Thus, the phrasing of the "info-gap robustness question" in the quoted paragraph conceals a great deal. It conceals the fact that the phrase

  "still guarantee that the outcome is acceptable"

  ought to be stated thus:

  "still guarantee that the $\boxed{\text{local worst-case}}$ outcome is acceptable"

  As we know by now, the fact that info-gap's robustness analysis amounts to a local worst-case analysis is manifested very clearly by the clause $r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})$ that appears both in info-gap's robustness model and in info-gap's robust-satisficing decision model.
- Take special note of the reference to situations where "... models can err enormously without preventing acceptable outcomes ...". Because, the implication is that in other situations info-gap decision theory is not capable of determining whether the system is robust or fragile against $\boxed{\text{severe}}$ uncertainty. The trouble with info-gap's determinations is that a system that info-gap's robustness model deems fragile is not necessarily fragile by measures of *global* robustness that assess the performance of the system over the entire uncertainty space. That is, a system that is fragile in the neighborhood of $\tilde{u}$ is not necessarily fragile elsewhere in the uncertainty space.

6.6 Unbounded uncertainty spaces

Consider again the following paragraph:

> These two concepts of robustness—min-max and info-gap—are different, motivated by different information available to the analyst. The min-max concept responds to severe uncertainty that nonetheless can be bounded. The info-gap concept responds to severe uncertainty that is unbounded or whose bound is unknown. It is not surprising that min-max and info-gap robustness analyses sometimes agree on their policy recommendations, and sometimes disagree, as has been discussed elsewhere.[(40)]
>
> Ben-Haim (2012, p. 7)

where reference [40] is Ben-Haim et al. (2009).

I want to take another look at this paragraph in order to focus more closely on the recurring emphasis in the info-gap literature on info-gap's much vaunted ability to handle unbounded uncertainty space.

- Again, in the attempt to sharply differentiate between info-gap robustness and of maximin-robustness, this paragraph not only conceals the fact that info-gap robustness is a simple *instance* of maximin-robustness but it gives a thoroughly distorted picture of maximin models. What is more, it gives an utterly distorted representation of info-gap's robustness model's mode of operation.
- To begin with, the claim that maximin models require their uncertainty space to be unbounded is **groundless.** There are many situations where the uncertainty space of a maximin model is unbounded (e.g. (2)).

- But what is more, as we know by now, if info-gap's robustness model can deal with unbounded uncertainty spaces then it goes without saying that so can prototype maximin models!
- But, it is particularly important to appreciate what info-gap's claimed ability to deal with unbounded uncertainty spaces **actually comes down to**. To be able to appreciate this fact it is important to keep in mind how info-gap's robust-satisficing decision model actually operates. Recall then that this model's **sole concern** is robustness with respect to a *constraint.* Thus, the results generated by this model are based only on whether for any pair $y = (q, \alpha)$, the constraint $r_c \leq r(q, u)$ is satisfied for all values of $U(\alpha, \tilde{u})$, or whether it is not. In other words, in its pursuit of a robust decision, this model's **primary concern** is not with whether the uncertainty space is bounded or unbounded but with whether a decision satisfies or violates the *constraint.* Which means that for all the fuss about the uncertainty space being unbounded, insofar as info-gap's robust-satisficing analysis is concerned, it is immaterial whether the space it operates in is bounded or unbounded. The bottom line is that each *neighborhood* $U(\alpha, \tilde{u}), \alpha \geq 0$ is *bounded.*
- I remind the reader that a *constraint driven* analysis is characteristic of maximin models whose objective functions are independent of the state variable. For instance, in the case of the maximin model (9), if $f(y, s) = h(y)$, then

$$\max_{y \in Y} \min_{s \in S(y)} \{f(y, s) : con(y, s), \forall s \in S(y)\} \rightarrow \max_{y \in Y} \min_{s \in S(y)} \{h(y) : con(y, s), \forall s \in S(y)\}$$
$$= \max_{y \in Y} \{h(y) : con(y, s), \forall s \in S(y)\}. \quad (58)$$

  Thus, the worst outcome for alternative y is equal to $h(y)$ if y is admissible, namely if it satisfies the constraints $con(y, s)$ for all $s \in S(y)$.

  In contrast, the worst-outcome for inadmissible values of y is catastrophic. So much so that they are excluded from the maximin model at the outset.

  Another angle from which to assess the real meaning of the assertion that "... The info-gap concept responds to severe uncertainty that is unbounded or whose bound is unknown ..."—an assertion which, needless to say, is made much of in Ben-Haim (2001, 2006, 2010)—is to take a look at Figure 5. This figure gives a far more realistic depiction of the consequences of info-gap's robustness model operating in an unbounded or for that matter a vast uncertainty space. The result of this model operating in an unbounded or vast space coupled with its analysis being, by definition, confined to the neighborhood the of $\tilde{u}$ is the *No Man's Land.* This means of course that the info-gap robustness of decision q takes no account whatsoever of the performance of decision q on the *No Man's Land.*

  One might well ask therefore: what is the big deal then in info-gap decision theory representing severe non-probabilistic uncertainty by unbounded uncertainty spaces if its "secret weapon" for dealing with the huge challenges posed by such spaces is to ... dodge these challenges altogether and address instead the question:

  - How robust is decision q to small deviations/perturbations in the value of $\tilde{u}$?

  Another important point—discussed in Sniedovich (2012)—is that all this hullabaloo about info-gap and its unbounded spaces actually reveals that info-gap scholars, and by implication *Risk Analysis* referees, confuse the following two facts:

Fact 6.1 *Info-gap allows its uncertainty space, $\mathscr{U}$, to be unbounded, hence it allows the neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$ to be vast (for large values of α). Thus, insofar as the representation of the neighborhoods is concerned, α is unbounded.*

Fact 6.2 *The imposition of the local worst-case robustness constraint* $r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})$ *on* α *implies that the* **admissible values** *of* α *pertaining to decision* q *are restricted to the range* $[0, \hat{\alpha}(q,r_c)]$. *In other words, within the framework of info-gap's robust-satisficing decision model the admissible value of* α *is* **bounded above** *by* $\hat{\alpha}(q,r_c)$.

What is so comical in all this is that info-gap scholars seem totally obtuse to the fact that, for all the fuss about α being unbounded, when it comes to determining robustness of decisions, this fact comes to naught. Because, to repeat, within the framework of info-gap's robust-satisficing decision model the admissible value of α is **bounded above** by $\hat{\alpha}(q,r_c)$.

And the immediate implication of this is that the info-gap robustness of decision q is determined in total disregard to the performance of decision q on the *No Man's Land* associated with this decision, namely $NML(q) := \mathscr{U} \setminus U(\alpha^*,\tilde{u})$, where $\alpha^* = \hat{\alpha}(q,r_c) + \varepsilon$.

Add to this the fact that according to Ben-Haim (2001, 2006, 2010), $\mathscr{U}$ is **typically** unbounded, the inevitable conclusion is that $NML(q)$ is equally **typically unbounded**. And the inference therefore must be that the info-gap robustness of decision q **typically** depends only on the performance of decision q over a **minute** (infinitesimal) neighborhood of $\mathscr{U}$ around $\tilde{u}$, as illustrated in Figure 5.

No amount of theoretic can change this fact.

As I have been arguing all along, a theory whose robustness model yields results that depend only on a minute (infinitesimal) neighborhood of the uncertainty space around a **wild guess** must be viewed as a **voodoo** decision theory.

- Just a few words about the contention that as a result of their purported different approaches to uncertainty, "... min-max and info-gap robustness analyses sometimes agree on their policy recommendations and sometimes disagree...".

  This, as this statement contends, was discussed in [40] (Ben-Haim et al. 2009). All I need to say here about the fact that the maximin model used in [40] (Ben-Haim et al. 2009) generates results that are different from the results generated by info-gap's robust-satisficing model only means this. In the article in question, info-gap's robust-satisficing model is compared to **an instance** of the prototype maximin model that yields results that are different from those of info-gap's robust-satisficing model. But, that instance of the maximin model specified according to Instance I in Figure 1 always generate exactly the same results as those generated by info-gap's robust-satisficing decision model.

- A review of reference [40] (Ben-Haim et al. 2009) is available on my website[8].

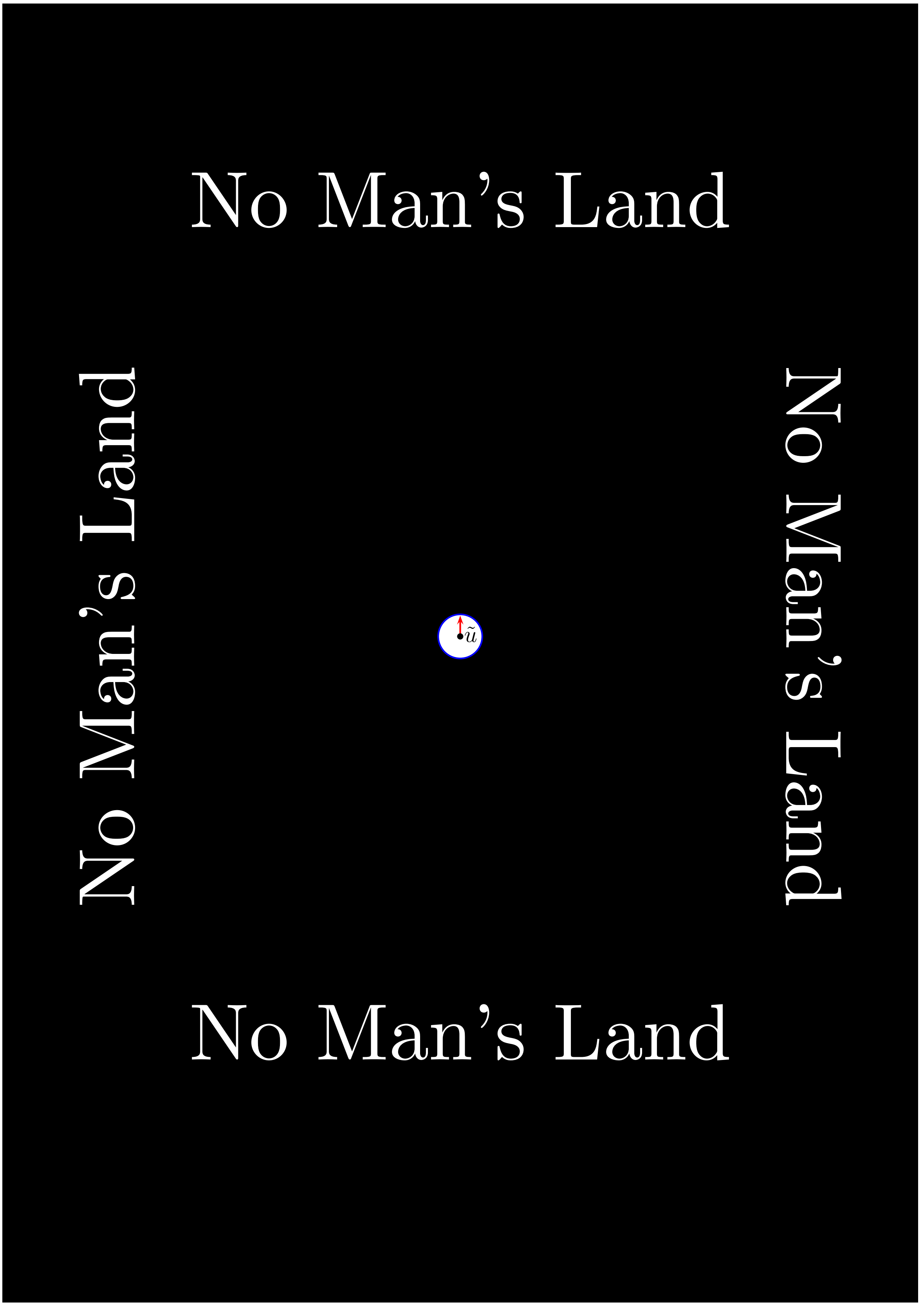
Remark

In a sequel to this article, I plan to take a closer look at the rhetoric in the info-gap literature, about the patently *too good to be true,* capability of info-gap's model of local robustness to provide a reliable tool for the management of a non-probabilistic severe uncertainty manifested in unbounded uncertainty spaces.

Here I give a few pointers to the misconceptions that are at the bottom of the claims denying that info-gap decision theory is "... a "local" theory of robustness ..." as exemplifies for instance in the following paragraph:

> If the robustness is not large, and especially if the robustness is small, then confidence is not warranted. If the robustness is small then confidence is warranted only "locally," near the models, while if the robustness is large then confidence is

[8]See http://info-gap.moshe-online.com/reviews/review_12.html



Warning: Given that the size of the page is finite and the uncertainty space is unbounded, it is necessary to exaggerate the size of the white circle. Take note then that the figure displays only a minute part of the *No Man's Land* as this *No Man's Land* extends limitlessly in all directions.

Figure 5: Info-gap's *No Man's Land* of decision q

warranted over a wide domain of deviation from the models. Info-gap theory uses the analyst's models, but this does not make it a "local" theory of robustness.

Ben-Haim (2012, p.)

My basic thesis is that claims such this stem from a profound confusion of the two facts I discuss above. Namely, the fact that info-gap decision theory allows its uncertainty space, $\mathscr{U}$ to be unbounded, and the fact that in the framework of info-gap's robust-satisficing decision model the admissible value of α is **bounded above** by $\hat{\alpha}(q, r_c)$. Incredible though it make sound, a basic confusion as to how these two facts relate to one another is apparently behind assertions such as: "... but this does not make it a "local" theory of robustness."

For consider what a denial of the fact that info-gap decision theory is "... a "local" theory of robustness" amounts to. This denial implies that in spite of the fact that info-gap decision theory's robustness model dictates that a *single* violation of the performance constraint $r_c \leq r(q, u)$ in the proximity of the estimate $\tilde{u}$ renders decision q fragile—regardless of how well/badly decision q performs elsewhere on $\mathscr{U}$—Ben-Haim (2012) insists that info-gap's robustness is not a measure of local robustness!

Happily, it is elementary to demonstrate the absurd in denying that info-gap decision theory is "... a "local" theory of robustness." For instance, it is easy to formulate examples where a decision is extremely robust over the uncertainty space $\mathscr{U}$, yet info-gap decision theory deems this decision to be very fragile. The pathologic case is, of course, where decision q satisfies the performance constraint everywhere on $\mathscr{U}$ except at $u = \tilde{u}$. The info-gap robustness of q is equal to 0, yet q is extremely robust against the variation of u over $\mathscr{U}$.

Another vivid illustration of the inherently local orientation of info-gap's robustness model is the crucial role that the point estimate $\tilde{u}$ plays in determining the robustness of decisions. This is illustrated in Figure 6, where the domains of acceptable values of u associated with two decisions, q' and q'', are represented by the respective shaded areas.

Note that the set of acceptable values of u associated with decision q' is markedly larger than the set of acceptable values of u associated with decision q''. Therefore, one can sensibly argue that decision q' is much more robust than decision q'' against variations in u over $\mathscr{U}$.

Yet, info-gap decision theory cannot assess the robustness of these decisions to determine which is the more robust. This is so because to determine the robustness of decisions, info-gap decision theory requires the analysts to specify the "center-point" of the locale, or neighborhood, where the robustness is assessed. The robustness would then depend on the location of this "center-point" in the uncertainty space $\mathscr{U}$. Since Figure 6 does not stipulate the location of this point, it is impossible to determine which decision is more info-gap robust in this case. Furthermore, to compare the info-gap robustness of the two decisions, we need to know how the neighborhoods around the "center-point" are defined.

In short, incredible as it may appear , based on the information displayed in this figure, info-gap decision theory cannot determine which decision is more robust against the severe uncertainty in the true value of u.

To resolve this difficulty, suppose that we allow the neighborhoods to be represented by circles centered at $\tilde{u}$ where "center-point" $\tilde{u}$ is located exactly at the center of $\mathscr{U}$, that is at the center of the white circle associated with decision q', which is also the center of the gray circle associated with decision q''.

In this case, the info-gap robustness of decision q' would be much smaller than the info-gap robustness of decision q''.

This, needless to say is not at all surprising because:

- Info-gap robustness is **not** a measure of robustness against the variations in the value of parameter u over its set of possible/plausible values, $\mathscr{U}$.
- Info-gap robustness is a measure of robustness against **small** deviations/perturbations in the value of the estimate $\tilde{u}$. The info-gap robustness of decision q is determined by the *local* performance of q in the neighborhood of the estimate. Large deviations from

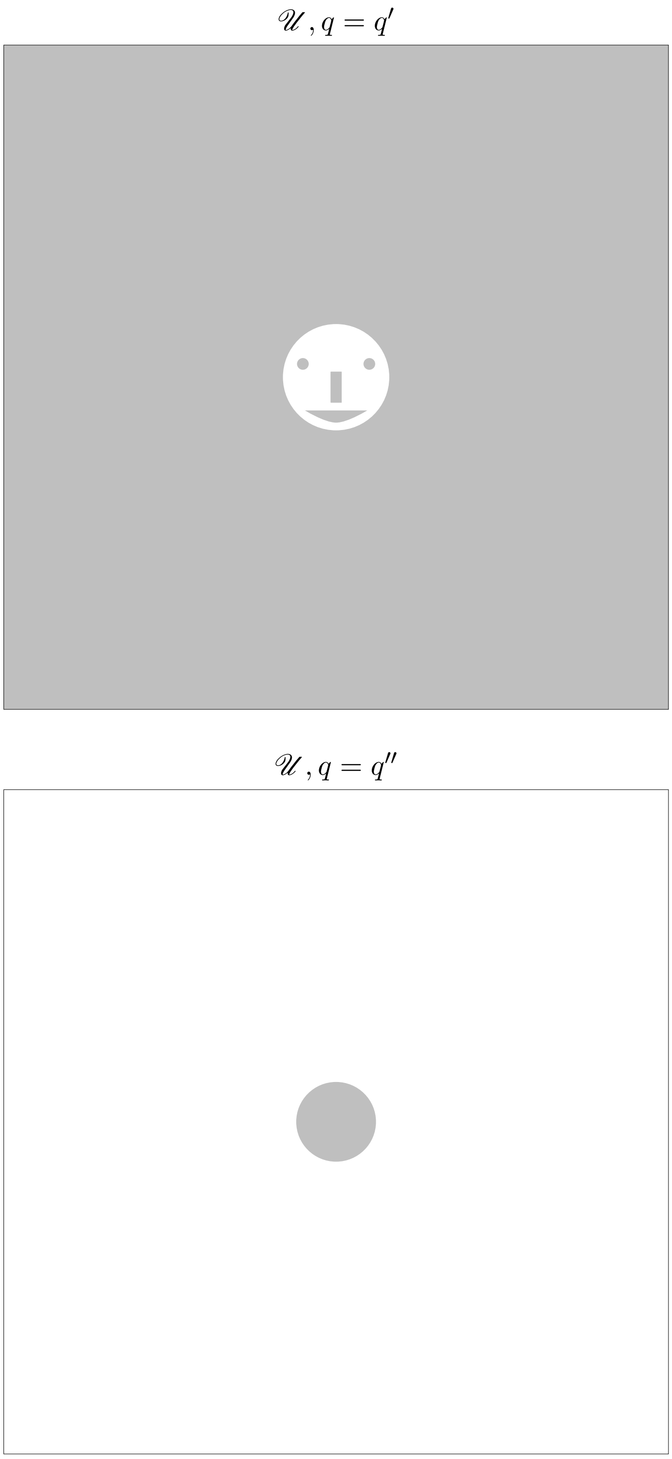


Figure 6: The tale of two decisions

the estimate are considered only if **all** smaller deviations do not violate the performance constraint.

This is a measure of *local* robustness par excellence.

The other side of the absurd assertion "... but this does not make it a "local" theory of robustness" is the attribution of "global" capabilities to info-gap's robustness model. To explain it, let

$$A(q) := \text{ set of acceptable values of } u \text{ pertaining to decision } q \tag{59}$$
$$= \{u \in \mathscr{U} : r_c \leq r(q,u)\}. \tag{60}$$

The staggering misconception that is exhibited in the attribution of "global" capabilities to info-gap's robustness model is evident in scores of info-gap publications. This misconception is manifested in the claims that info-gap decision theory ranks decisions according to the size of their sets of acceptable values of u, such that the larger the better, hence that it selects the decision with the largest set of acceptable values of u. For instance,

> Info-gap analysis allows the decision maker to identify solutions that perform satisfactorily well under the widest possible range of conditions.
>
> Hall and Ben-Haim (2007, p. 7)

> The robust satisficer answers two questions: first, what will be a "good enough" or satisfactory outcome; and second, of the options that will produce a good enough outcome, which one will do so under the widest range of possible future states of the world.
>
> Schwartz, Ben-Haim and Dasco (2010, p. 213)

> It asks, instead, "What kind of return do we want in the coming year, say, in order to compare favorably with the competition? And what strategy will get us that return under the widest array of circumstances?"
>
> Schwartz, Ben-Haim and Dasco (2010, p. 220)

> For an individual who recognizes the costliness of decision making, and who identifies adequate (as opposed to extreme) gains that must be attained, a satisficing approach will achieve those gains for the widest range of contingencies.
>
> Schwartz, Ben-Haim and Dasco (2010, p. 223)

> Decisions that cause the system to exceed the performance criterion over a wide range of uncertainty are said to be more "robust" or "immune to failure" (Ben-Haim 2006).
>
> van der Burg and Tyre (2011, 304)

To show the absurd in these claims is straightforward. For consider Figure 7, where the shaded areas represent the sets of acceptable values of u pertaining to the two decisions, namely $A(q')$ and $A(q'')$. Note that in spite of the fact that set $A(q')$ is much larger than set $A(q'')$, the info-gap robustness of decision q'' is deemed, according to the precepts of info-gap decision theory, larger than the info-gap robustness of decision q'.

In short: info-gap's robustness of decision q does not measure the "size" of the set of acceptable values of u associated with decision q. Hence, info-gap's robust-satisficing decision model does not seek a decision whose set of acceptable values of u is the largest. Info-gap robustness is not a measure of the "size" of $A(q)$, it is a measure of the "size" of the largest *neighborhood around* $\tilde{u}$ that is contained in $A(q)$. This, obviously, is a measure of *local* robustness.

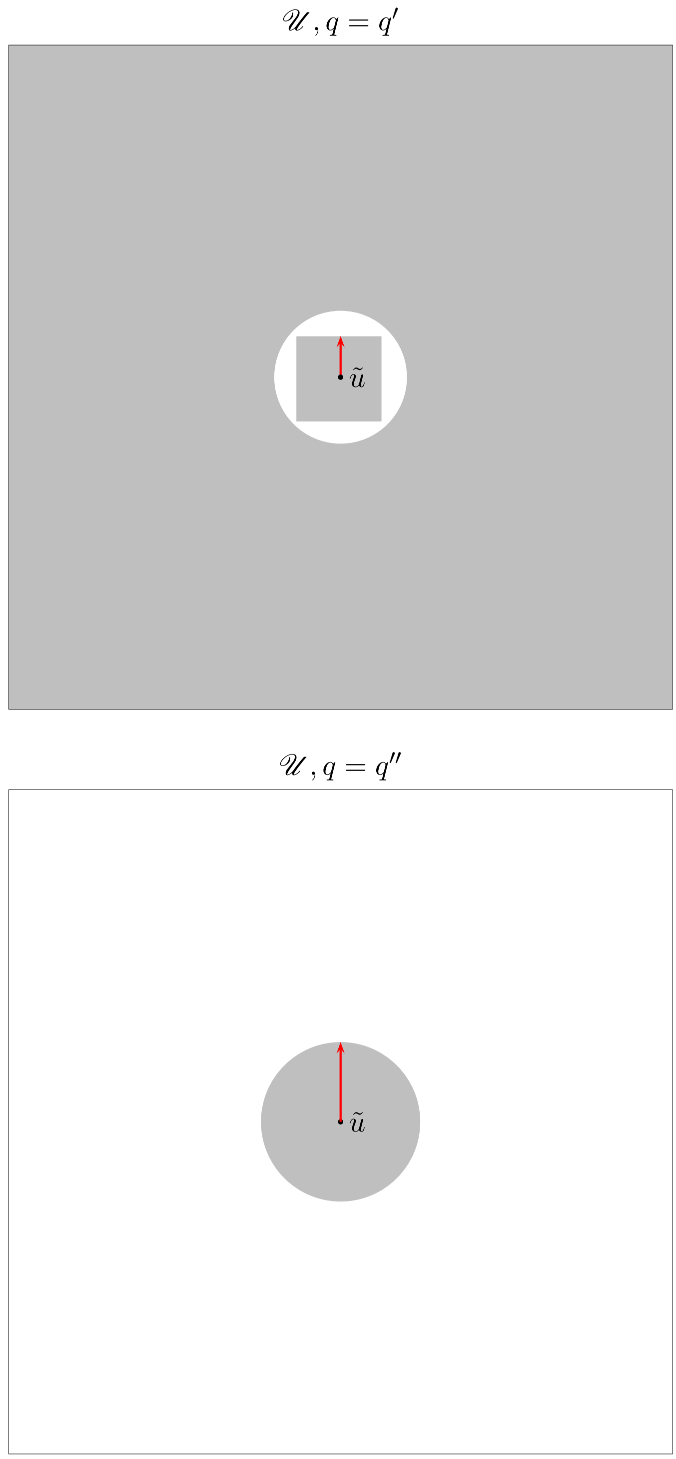


Figure 7: The tale of two decisions

6.7 What does all of this come down to?

The discussion in Ben-Haim (2012, pp. 6-7) on the differences and similarities between maximin models and info-gap's robust-satisficing decision model is grossly misleading. It depicts info-gap decision theory as a theory that was designed specifically to deal with situations that cannot be handled by maximin models due to the extreme severity of the uncertainty: the uncertainty space is unbounded or its bound is unknown. It thus misleadingly suggests that info-gap's robust-satisficing decision model can deal with situations that are outside the purview of maximin models.

The discussion is also problematic technically:

- It erroneously suggests that maximin models requires their *uncertainty space* to be bounded.
- It misleadingly suggests that the specification of info-gap's uncertainty space $\mathscr{U}$ is somehow fundamentally different from the specification of the *state space* of maximin models.
- It misleadingly suggests that maximin models cannot address robustness against deviations from a nominal value of a parameter.

It is particularly noteworthy that the discussion is completely oblivious to the difference between *local* and *global* robustness and robustness with respect to *payoffs* vs robustness with respect to *constraints.* These differences are crucial for a full appreciation of the relationship between generic maximin models such as (9) and the instance used in info-gap decision theory, namely (8).

In short, the discussion in Ben-Haim (2012, pp. 6-7) gives an utterly distorted account of the role and place of info-gap decision theory in decision making under severe uncertainty. As elsewhere in the info-gap literature, the misconceptions in Ben-Haim's (2012, pp. 6-7) stem from a comparison of the pair

Maximin model	Info-gap's robust-satisficing decision model
$\max\limits_{q\in Q} \min\limits_{u\in U(\alpha',\tilde{u})} r(q,u)$	$\max\limits_{q\in Q,\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$

(61)

for some given value of α'.

The following question is therefore inescapable: why the reluctance to discuss the similarities and differences between these two models?

(The Prototype) Maximin model	(The Instance that Roared) Info-gap's robust-satisficing decision model
$\max\limits_{y\in Y} \min\limits_{s\in S(y)} \{f(y,s) : con(y,s), \forall s \in S(y)\}$	$\max\limits_{q\in Q,\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$

(62)

Is it because such a comparison will immediately confirm that info-gap's robust-satisficing decision model is indeed a maximin model after all, and thus will expose the rhetoric in the info-gap literature for what they are?

As for the worst case issue:

The narrative in the info-gap literature on the non-existence of a worst case in unbounded uncertainty spaces and the alleged secret weapon possessed by info-gap decision theory to deal with unbounded uncertainty spaces amounts to no more than a misleading rhetoric.

This narrative conceals the following simple hard fact:

Fact 6.3 *Info-gap's robustness model and info-gap's robust-satisficing model belong in a class of maximin models whose objective function is independent of the uncertainty parameter (state variable). Such models seek robustness only with respect to constraints. Hence, in the context of these models the existence of a worst case is a not an issue. This is so because, for each alternative, a worst outcome always exists even if the uncertainty space is unbounded.*

As an aside, readers who are familiar with the concept *Radius of Stability* (circa 1960) are advised that info-gap's robustness model is a simple radius of stability model (Sniedovich 2010, 2012, 2012a, 2012b).

7 Summary and conclusions

Info-gap decision theory is based on two simple mathematical models whose basic properties are transparent so as to require no elaborate analysis/investigation:

Info-gap's robustness model	Info-gap's robust-satisficing decision model
$\max\limits_{\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$	$\max\limits_{q\in Q,\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$

(63)

It is elementary to prove formally and rigorously that both are instances of the following prototype maximin model:

$$\max_{y\in Y} \min_{s\in S(y)} \{f(y,s) : con(y,s), \forall s \in S(y)\}. \tag{64}$$

Such proofs have been in the public domain since 2007.

No amount of rhetoric can change these facts.

And yet!

Respectable peer-reviewed journals, such as *Risk Analysis*, continue to publish articles asserting fallacies both about Wald's maximin paradigm and info-gap decision theory, and the relationship between them.

It would seem that this is due to the rhetoric that info-gap scholars use as a substitute for a rigorous analysis of the above three models and their relationships, which conceal from referees the hard facts!

I submit that the readership of these journals deserve better!

Challenging risk analysis problems, such as decision under a severe, non-probabilistic uncertainty with unbounded uncertainty spaces, cannot be "solved" by rhetoric about a model of local robustness. Neither can a well-established 40-year old model of local robustness become new and radically different at a stroke of a pen. Such rhetoric impedes rather than facilitates progress in this important and challenging area of risk analysis.

Followers of info-gap decision theory, indeed readers of the info-gap literature, would therefore be well advised to assess critically misleading statements such as these:

> Info-gap decision theory is radically different from all current theories of decision under uncertainty. The difference originates in the modeling of uncertainty as an information gap rather than as a probability. The need for info-gap modeling and management of uncertainty arises in dealing with severe lack of information and highly unstructured uncertainty.
>
> Ben-Haim (2001, 2006, p. xii)

> In this book we concentrate on the fairly new concept of information-gap uncertainty, whose differences from more classical approaches to uncertainty are real and deep. Despite the power of classical decision theories, in many areas such as engineering, economics, management, medicine and public policy, a need has arisen for a different format for decisions based on severely uncertain evidence.
>
> Ben-Haim (2001, 2006, p. 11)

> Probability and info-gap modelling each emerged as a struggle between rival intellectual schools. Some philosophers of science tended to evaluate the info-gap approach

in terms of how it would serve physical science in place of probability. This is like asking how probability would have served scholastic demonstrative reasoning in the place of Aristotelian logic; the answer: not at all. But then, probability arose from challenges different from those faced the scholastics, just as the info-gap decision theory which we will develop in this book aims to meet new challenges.

Ben-Haim (2001 and 2006, p. 12)

The emergence of info-gap decision theory as a viable alternative to probabilistic methods helps to reconcile Knights dichotomy between risk and uncertainty. But more than that, while info-gap models of severe lack of information serve to quantify Knights unmeasurable uncertainty, they also provide new insight into risk, gambling, and the entire pantheon of classical probabilistic explanations. We realize the full potential of the new theory when we see that it provides new ways of thinking about old problems.

Ben-Haim (2001 p. 304; 2006, p. 342)

Info-gap decision theory clearly presents a 'replacement theory' with which we can more fully understand the relation between classical theories of uncertainty and uncertain phenomena themselves.

Ben-Haim (2001 p. 305; 2006, p. 343)

The management of surprises is central to the "economic problem", and info-gap theory is a response to this challenge. This book is about how to formulate and evaluate economic decisions under severe uncertainty. The book demonstrates, through numerous examples, the info-gap methodology for reliably managing uncertainty in economics policy analysis and decision making.

Ben-Haim (2010, p. x)

Info-gap theory is not a worst case analysis. While there may be a worst case, one cannot know what it is and one should not base one's policy upon guesses of what it might be.

Ben-Haim (2010, p. 9)

The difference from min-max approaches is that we are able to select a policy without ever specifying how wrong the model actually is. Min-max and info-gap robust-satisficing strategies will sometimes agree and sometimes differ.

Ben-Haim (2010, p. 10)

The info-gap model is unbounded in the sense that there is no largest set and there is no worst case.

Carmel and Ben-Haim (2005, p. 635)

It is important to emphasize that the robustness $\hat{h}(R_*, c)$ is *not* a minimax algorithm. In minimax robustness analysis, one *minimizes* the *maximum* adversity. This is not what info-gap robustness does. There is no maximal adversity in an info-gap model of uncertainty: the worst case at any horizon of uncertainty h is less damaging than some realization at a greater horizon of uncertainty. Since the horizon of of uncertainty is unbounded, there is no worst case and the info-gap analysis cannot and does not purport to ameliorate a worst case.

Ben-Haim (2005, p. 392)

While there is a superficial similarity with minimax decision making, no fixed bounds are imposed on the set of possibilities, leading to a comprehensive search of the set of possibilities and construction of functions that describe the results of that search.

Hine and Hall (2010, p. 17)

> Info-gap generalizes the maximin strategy by identifying worst-case outcomes at increasing levels (horizons) of uncertainty. This permits the construction of 'robustness curves' that describe the decay in guaranteed minimum performance (or worst-case outcome) as uncertainty increases.
>
> Wintle et al. (2011, p. 357)

> These two concepts of robustness—min-max and info-gap—are different, motivated by different information available to the analyst. The min-max concept responds to severe uncertainty that nonetheless can be bounded. The info-gap concept responds to severe uncertainty that is unbounded or whose bound is unknown. It is not surprising that min-max and info-gap robustness analyses sometimes agree on their policy recommendations, and sometimes disagree, as has been discussed elsewhere.(40)
>
> Ben-Haim (2012, p. 7)
>
> (40) = Ben-Haim et al. (2009).

> The relation between min-max and info-gap robust-satisficing has been discussed at length elsewhere.[12] The two methods have much apparent similarity, though also important differences. Most significantly, they depend on different prior information, and can lead to different solutions. Briefly, min-max requires knowledge of a worst case. In contrast, the horizon of uncertainty of an info-gap model is unknown and unbounded, thus deliberately avoiding the specification of a worst case. On the other hand, the info-gap robustness does require the analyst to specify the worst acceptable outcome, which in engineering design is usually a design specification.
>
> Ben-Haim (2012a, p. 9)
>
> [12] = Ben-Haim et al. (2009).

Experience with info-gap decision theory suggests that referees of journals devoted to the study of risk, such as *Risk Analysis,* should be more vigilant about the claims regarding the nature of this theory, and the assertion about its place in the state of the art.

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