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What's next on the *Risk Analysis/Info-Gap* agenda?

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Abstract

The discourse on info-gap decision theory in the journal *Risk Analysis* continues to yield groundbreaking results. The latest such result claims, among other things, that a real-valued function cannot be optimized, in a global sense, on an unbounded domain! For instance, according to this latest development, the value of $sin(x)$ cannot be optimized (in a global sense) over the real line, thus contradicting the long established mathematical fact that infinitely many global optima exit in this case. Given this state of affairs, it is only natural to ask: what's next on the *Risk Analysis/Info-Gap* agenda? In this short discussion I speculate on this intriguing question.

Keywords: info-gap decision theory, maximin, peer-review, rhetoric, risk analysis, unconstrained optimization, worst-case analysis.

1 The latest development

An article on info-gap decision theory that was published recently in the journal *Risk Analysis,* argues as follows (emphasis added):

> These two concepts of robustness–min-max and info-gap–are different, motivated by different information available to the analyst. **The min-max concept responds to severe uncertainty that nonetheless can be bounded. The info-gap concept responds to severe uncertainty that is unbounded or whose bound is unknown.** It is not surprising that min-max and info-gap robustness analyses sometimes agree on their policy recommendations, and sometimes disagree, as has been discussed elsewhere.$^{(40)}$
>
> Ben-Haim (2012, p. 1644)[1]

where reference (40) is Ben-Haim et al. (2009).

Arguments of this type have been bandied about in publications on info-gap decision theory ever since the theory's inception. Their objective is to back up the claim that info-gap robustness is not a worst-case robustness, hence that info-gap's robust-satisficing decision model is not a maximin model (see for example, Ben-Haim 2001, 2006, 2007, 2010, 2012).

But this is the first time that such an **erroneous** claim is made in an article published in the journal *Risk Analysis,* henceforth *Journal.* What is so surprising in all this is that this argument passed muster in the

[1]The complete text discussing this issue in Ben-Haim (2012), is quoted in section 8.

Journal's peer-review process. It is surprising because one assumes that referees of the *Journal* assigned to review this article should know that not only do such claims lack any merit, they are in fact ... **absurd**.

Indeed, the fact that this argument passed muster is doubly surprising because, in the article "*Fooled by local robustness*", which was published in the same issue of the *Journal,* Sniedovich (2012a) proves formally and rigorously that info-gap's robustness model is a very simple maximin model[2]. As a matter of fact, info-gap robustness is a re-invention of a concept that is universally known as **radius of stability** (circa 1960)[3]. So the question is this: how is it that maximin models require the uncertainty parameter to be bounded, when info-gap's robustness model, which is a very simple maximin model, can handle an unbounded uncertainty?

One of the main objectives of this short article is then to call the attention of the referees and area editor who accepted Ben-Haim (2012) for publication in the *Journal,* to the misguided reasoning behind the claims in Ben-Haim (2012), and to remind them of the true nature of the relationship between info-gap robustness and maximin robustness.

2 Optimizing over the real line

You will recall that a "textbook" *minimax model* (e.g. Demyanov and Malozemov 1990, Du and Pardalos, 1995) conducts its worst-case analysis (Rustem and Howe, 2002) by *maximizing* a real-valued function on the model's state (uncertainty) space. Thus, the immediate implication of the statement quoted above from Ben-Haim (2012) is that a real-valued function cannot be *maximized* (in a global sense) on an unbounded domain. Ben-Haim's (2012) claim is therefore disconcerting, because if, for simplicity, we assume that the state (uncertainty) space is the real line $\mathbb{R} = (-\infty, \infty)$, it would follow that, according to Ben-Haim (2102), we cannot *maximize* (in a global sense) a real-valued function on the real line!!! A similar argument for the *maximin model* would imply that we cannot *minimize* (in a global global sense) a real-valued function on the real-line!!!

This new development is certainly most alarming for the old stalwart **unconstrained optimization**, because it rules out the global *unconstrained optimization* of a real-valued function on the real line!!!

In short, to show how absurd Ben-Haim's (2012) claim is, consider the following abstract (global) *optimization problem*

$$z^* := \min_{y \in Y} \ f(y) \tag{1}$$

where Y is some set and f is a real-valued function on Y.

What Ben-Haim's (2012) claim implies, among other things, is that for this global optimization problem to have an optimal solution, set Y must be **bounded.**

3 But ...

Examining the material taught in first year university/college mathematics, we see that optimizing (in a global sense) real-valued functions on the real line is often done as a matter of course. For example, consider the case where $Y = \mathbb{R}$ and $f(y) = sin(y)$. Clearly, there are infinitely many global optima in this case. And in the case of $f(y) = y^2$, clearly $y^* = 0$ is a global minimum. And how about optimizing $(y-4)/(y^2+5)$ over $y \in \mathbb{R}$?

The inference must therefore be that this new development can mean only one thing: either introductory calculus textbooks must be ditched forthwith. Or, Ben-Haim's (2012) latest claim is absurd.

[2] Such proofs have been available at least since 2007, e.g. Sniedovich (2007, 2008, 2010, 2012).

[3] See formal rigorous proofs in Sniedovich (2010, 2012, 2012a).

4 An obvious counter example

For the benefit of readers who may not see how the discussion in the preceding section on global optimization over unbounded domains brings out the absurd in Ben-Haim's (2012) claim, consider this unbounded *minimax problem:*

$$z^{**} := \min_{-\infty<x<\infty} \overbrace{\max_{-\infty<y<\infty} \{x^2 + 2xy - y^2\}}^{\text{worst-case analysis}} . \tag{2}$$

Note that for any given value of x, the optimal value of y, namely the maximizer of $x^2 + 2xy - y^2$ over $y \in (-\infty, \infty)$, is equal to $y(x) = x$. Hence, the optimal value of x is the minimizer of $x^2 + 2xy(x) - y^2(x) = 2x^2$ over $x \in (-\infty, \infty)$, namely $x^* = 0$. In short, the optimal solution is $(x^*, y^*) = (0, 0)$, yielding $z^{**} = 0$. Contrary to Ben-Haim's (2012) claim, this unbounded minimax problem most definitely has an optimal solution.

5 The Error

Consider now the textbook minimax model:

$$z^{\circ} := \min_{x\in X} \max_{s\in S} g(x, s) \tag{3}$$

where X denotes the set of *alternatives,* S denotes the *state space,* and $g(x, s)$ denotes the *outcome* generated by alternative x and state s. I shall refer to the state space S also as the *uncertainty space*.

Keep in mind that according to Ben-Haim (2012), for this minimax model to be applicable, the state (uncertainty) space S must be **bounded.** Keep also in mind that, in the framework of this minimax model, the worst-case analysis is conducted by the inner $\max_{s\in S}$ operation. So let

$$wo(x) := \max_{s\in S} g(x, s) \ , \ x \in X \tag{4}$$

denote the **worst outcome** associated with alternative x, often called the **security level** of alternative x (Resnik 1987, French 1988).

Ben-Haim (2012) asserts then that for the worst outcomes $wo(x), x \in X$ to exist, the state (uncertainty) space S must be **bounded**. To see why this claim is without any foundation, let us trace out the misguided reasoning behind it. Here is a schema of this reasoning:

Misguided reasoning

- **Step 1.** For the minimax model (3) to be applicable, the worst outcomes $wo(x), x \in X$ must exist.
- **Step 2.** For a worst outcome to exit for alternative x, the outcome set $g(x, S) := \{g(x, s); s \in S\}$ must be bounded above.
- **Step 3.** For the outcome set $g(x, S) := \{g(x, s); s \in S\}$ to be bounded above, the uncertainty space S must be bounded.
- **Step 4.** Hence, for the minimax model to be applicable, the state (uncertainty) space S must be bounded.

The blunder is in **Step 3.**

The uncertainty space S most definitely **need not** be bounded for the outcome sets $g(x, S), x \in X$, to be bounded above. For instance, in the case of the minimax model specified in (2), we have $g(x, s) = x^2 + 2xs - s^2$ and $S = (-\infty, \infty)$, hence

$$g(x, S) : = \left\{x^2 + 2xs - s^2 : -\infty < s < \infty\right\} \tag{5}$$

$$= \left(-\infty, 2x^2\right] \ , \ -\infty < x < \infty. \tag{6}$$

This means that although S is unbounded, for each alternative $x \in X = (-\infty, \infty)$, the set of outcomes $g(x, S) = \big(-\infty, 2x^2\big]$ is bounded above.

It is important to point out that the misguided reasoning described above exhibits not only a profound misunderstanding of the characteristics of a minimax model's unbounded state (uncertainty) space, but also of that of a bounded space. The point to note here is that a bounded state (uncertainty) space S by itself does not assure that the worst outcomes $wo(x), x \in X$, do indeed exist. For instance, consider the simple instance of (3) where $X = [1, 2]$, $S = [-1, 1]$ and

$$g(x, s) = \begin{cases} x + s & , \ s \in [-1, 0] \\ x + s^{-2} & , \ s \in (0, 1] \end{cases} \ , \ x \in [1, 2], s \in [-1, 1]. \tag{7}$$

Observe that even though the state (uncertainty) space $S = [-1, 1]$ is bounded, for each $x \in X$ the outcome space $g(x, S)$ is unbounded above, hence there are no worst outcomes.

In a nutshell, claims that the existence of a worst case is contingent on the state (uncertainty) space being bounded are doubly in error: (i) a bounded state (uncertainty) space is not a sufficient condition for the existence of worst outcomes; and (ii) an unbounded state (uncertainty) space does not rule out the existence of worst outcomes.

6 No spin zone

Info-gap's robust-satisficing approach to severe uncertainty is based on two simple **mathematical models** whose structures are transparently clear. Therefore, the only meaningful way to clarify the relationship between info-gap's robust-satisficing approach to severe uncertainty and Wald's maximin paradigm, is to compare the **mathematical models** that info-gap decision theory deploys to this end, with generic mathematical models representing Wald's maximin paradigm. So, let us set off info-gap's core models against a generic maximin model and let . . . the **mathematics** do the talking.

Consider then the two core models of info-gap's robust-satisficing approach to severe uncertainty (Ben-Haim 2001, 2006, 2010):

Info-gap's robustness model	Info-gap's robust-satisficing decision model
$\hat{\alpha}(q, \tilde{u}) := \max\limits_{\alpha \geq 0} \{\alpha : r_c \leq r(q, u), \forall u \in U(\alpha, \tilde{u})\}$	$\hat{\alpha}(\tilde{u}) := \max\limits_{q \in Q, \alpha \geq 0} \{\alpha : r_c \leq r(q, u), \forall u \in U(\alpha, \tilde{u})\}$

recalling that $\tilde{u}$ denotes a point estimate of the true value of the uncertainty parameter u and $U(\alpha, \tilde{u})$ denotes a neighborhood of size α around $\tilde{u}$. Next, compare these two models to this generic maximin model:

$$z^* := \max_{x \in X} \min_{s \in S(x)} \{f(x, s) : con(x, s), \forall s \in S(x)\} \tag{8}$$

where

$$X = \text{ set of } \textit{alternatives} \text{ available to the decision maker.} \tag{9}$$

$$S(x) = \text{ set of } \textit{states} \text{ associated with alternative } x \in X. \tag{10}$$

$$con(x, s) = \text{ list of } \textit{constraints} \text{ imposed on the } (x, s) \text{ pairs.} \tag{11}$$

$$f(x, s) = \textit{payoff/reward} \text{ generated by alternative } x \text{ and state } s. \tag{12}$$

All we have to do to prove formally and rigorously that info-gap decision theory's two core models are indeed maximin models, is to identify the two instances of the generic maximin model (8) that correspond to these core models. So consider the two simple instances of (8) displayed in Figure 1.

THEOREM 1 *Info-gap's robustness model and info-gap's robust-satisficing decision model are simple maximin models. Specifically, both are simple instances of the generic maximin model given in (8).*

Generic maximin object in (8)	Instance I	Instance II
x	α	(q,α)
s	u	u
X	$[0,\infty)$	$Q\times[0,\infty)$
$S(x)$	$U(\alpha,\tilde{u})$	$U(\alpha,\tilde{u})$
$f(x,s)$	α	α
$con(x,s)$	$r_c \leq r(q,u)$	$r_c \leq r(q,u)$

Note: in *Instance I* the objects q and $\tilde{u}$ are fixed and given, and in *Instance II* the object $\tilde{u}$ is fixed and given.

Figure 1: Two instances of the generic maximin model (8)

Proof. Substituting the specification of *Instance I* in the maximin model (8) yields the following simple maximin model:

$$z^* := \max_{x\in X} \min_{s\in S(x)} \{f(x,s) : con(x,s), \forall s\in S(x)\} \tag{13}$$

$$= \max_{\alpha\geq 0} \min_{u\in U(\alpha,\tilde{u})} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\} \tag{14}$$

$$= \max_{\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u\in U(\alpha,\tilde{u})\}. \tag{15}$$

This is none other than info-gap's robustness model. And repeating the exercise with *Instance II*, yields the following simple maximin model:

$$z^* := \max_{x\in X} \min_{s\in S(x)} \{f(x,s) : con(x,s), \forall s\in S(x)\} \tag{16}$$

$$= \max_{q\in Q,\alpha\geq 0} \min_{u\in U(\alpha,\tilde{u})} \{\alpha : r_c \leq r(q,u), \forall u\in U(\alpha,\tilde{u})\} \tag{17}$$

$$= \max_{q\in Q,\alpha\geq 0} \{\alpha : r_c \leq r(q,u), \forall u\in U(\alpha,\tilde{u})\} \tag{18}$$

which is none other than info-gap's robust-satisficing decision model. **QED**

7 The instance that roared

It is important to appreciate that info-gap decision theory's two core models are not just maximin models. They are **very simple** maximin models. This is so because both models seek robustness **only** with respect to a performance **constraint,** meaning that for each alternative, the value of the *payoff/reward function* is **fixed** in the worst-case analysis. Indeed, the payoff/reward function f itself has a truly simple form: it is equal to α. And what is more, the state (uncertainty) space associated with alternative α is a **bounded neighborhood** of size α around a nominal value of the uncertainty parameter.

By analogy, consider a generic *polynomial* of the form

$$p(x) := a_0 + a_1x + a_2x^2 + \cdots + a_nx^n \ , \ -\infty < x < \infty \tag{19}$$

where the parameters $a_0,\dots,a_n$ are numeric scalars, and the following simple instance thereof, where the parameters A and B are numeric scalars:

$$L(x) := (x-A)(x-B) \ , \ -\infty < x < \infty. \tag{20}$$

Can you imagine claims in a peer-reviewed journal contending that this simple instance of (19) has capabilities that the prototype (19) lacks?

And yet, this is precisely the kind of claim that is made in Ben-Haim (2012) about Wald's maximin paradigm, which has the inherent ability to provide an array of maximin models that are immeasurably more powerful than info-gap's robustness model and info-gap's robust-satisficing decision model, e.g. the generic (8). So, to put across the profound absurd in the claims that a very simple instance of a prototype has capabilities that the prototype lacks, Sniedovich (2012e) dubs info-gap's robustness model the **instance that roared**. And to see how appropriate this label is, consider the following assertions:

> These two concepts of robustness–min-max and info-gap–are different, motivated by different information available to the analyst.
>
> Ben-Haim (2012, 1644)

> It is not surprising that min-max and info-gap robustness analyses sometimes agree on their policy recommendations, and sometimes disagree, as has been discussed elsewhere.$^{(40)}$
>
> Ben-Haim (2012, 1644)

The point to note about the latter assertion is this. By the same token that (as shown above) it is straightforward to formulate from the generic maximin model (8) maximin models that are totally identical to info-gap's robustness model and info-gap's robust-satisficing decision model; it is equally straightforward to obtain from it maximin models that are related to info-gap's robust-satisficing decision model, but are different from it, not to mention maximin models that are radically different from info-gap decision theory's two core models. After all, this is precisely the nature of the relationship between a **prototype** and its very **simple instances.**

For the record, the maximin model alluded to in reference (40) in Ben-Haim (2012) is a case in point. This model is as follows:

$$\max_{q\in Q} \min_{u\in U(\alpha',\tilde{u})} r(q,u) \tag{21}$$

where $\alpha' \geq 0$ is fixed and given.

Clearly, although this maximin model is related to info-gap's robust-satisficing decision model, which is another maximin model, is different from it. Unsurprisingly therefore, each model may yield different results.

In short, the fact that info-gap's robust-satisficing decision model is dissimilar from (21) does not alter the fact that info-gap's robustness model and info-gap's robust-satisficing decision model are simple maximin models, indeed simple instances of (8).

No amount of rhetoric can change this.

8 The art of rhetoric

As we saw above, it is straightforward to prove formally and rigorously that the claim that info-gap's robustness model is not a maximin model, is groundless. We also saw that it is straightforward to show that the argument purporting to back up this claim is absurd. The question therefore is: how could these claims pass muster through the *Journal*'s review process? Because, the question that the *Journal* must answer is this:

- How can a **simple instance** of a **prototype** model possibly be able to perform feats that the **prototype** model is unable to?!

The answer apparently is this:

Never, ever, underestimate the power of empty rhetoric!

And to illustrate, compare the above theorem to the following peer-reviewed rigmarole:

> **3.4. Robustness and Worst Cases: Two Approaches**
>
> There are many types of risk analysis partly because ignorance and uncertainty come in many forms. Probabilistic uncertainty induces probabilistic risk analysis, while starker uncertainty—for instance, ignorance of relevant probability distributions—engenders other analyses of risk.

A widely occurring operational distinction between risk analyses hinges on whether or not meaningful worst cases can be identified. When one can plausibly specify the worst events that can occur (and presuming we don't know probability distributions), then one might justifiably try to ameliorate these worst contingencies. This can be done in many different ways, and we will refer to this type of strategy as min-max analysis: minimizing the maximum damage.

The ability to implement a min-max analysis depends on identifying meaningful worst contingencies. This is feasible in many situations. The concept of a "meaningful worst case" depends on knowledge and judgment that may be within the risk analyst's competence. However, it is not usually sufficient to specify a worst case in some formal or abstract sense, such as the set of all contingencies that are consistent with the laws of science. A min-max analysis based on such an inclusive formulation may be uselessly overconservative. Min-max analysis is most useful when the analyst is able to avoid vacuous specification of worst cases. However, when information is really scarce, for instance, when processes are poorly understood or changing, then even typical cases cannot be reliably identified. It may then be impossible to meaningfully specify the boundary between extreme but possible occurrences, and the impossible or negligible.

Nonetheless, even when worst cases cannot be meaningfully specified, the analyst still has data, understanding, and mathematical representations: models in the broad sense that we are using that term. It is simply that the analyst cannot responsibly specify the magnitude of error of these models. For instance, we have many models for long-range climate change, but the earnest scientific disputes over these models may preclude the ability to confidently bound the errors. Or, introducing a new species to an ecosystem, either deliberately as an genetically modified organism or inadvertently by invasion, may alter the ecosystem dynamics in unknown ways.

In such situations one can still formulate and implement a robustness analysis. Info-gap theory has been developed precisely for the task. Let's discuss min-max and info-gap concepts of robustness.

The min-max concept of robustness responds to the question: How bad is the worst case? This is valuable information for the risk analyst and decisionmaker because if the worst case—after amelioration by a min-max analysis—is tolerable, then one can reasonably say that the system is robust to uncertainty.

The info-gap concept of robustness responds to a different question: How wrong can the models be and still guarantee that the outcome is acceptable? This is useful for the risk analyst and decisionmaker because if the models can err enormously without preventing acceptable outcomes, then one can reasonably say that the system is robust to uncertainty.

These two concepts of robustness—min-max and info-gap—are different, motivated by different information available to the analyst. The min-max concept responds to severe uncertainty that nonetheless can be bounded. The info-gap concept responds to severe uncertainty that is unbounded or whose bound is unknown. It is not surprising that min-max and info-gap robustness analyses sometimes agree on their policy recommendations, and sometimes disagree, as has been discussed elsewhere.[(40)]

Ben-Haim (2012, pp. 1643-1644)

Note: citation (40) in Ben-Haim (2012), refers to Ben-Haim et al. (2009, pp. 1061-1062), which puts forth an even more prolix, hence more seriously flawed, comparison of info-gap robustness and maximin robustness. A review of Ben-Haim et al. (2009) can be found elsewhere[4].

A detailed anatomy of this misguided rhetoric can be found in Sniedovich (2012e). For now, I urge readers to reread the theorem and to compare it, again, to the hollow rhetoric in Ben-Haim (2012), quoted above. And if you wonder what purpose does this rhetoric serve, take note that it is required to back up an even more extravagant rhetoric (emphasis added):

Info-gap decision theory is radically different from all current theories of decision under uncertainty. The difference originates in the modeling of uncertainty as an information gap rather

[4] See http://info-gap.moshe-online.com/reviews/review_12.html

than as a probability. The need for info-gap modeling and management of uncertainty arises in dealing with severe lack of information and highly unstructured uncertainty.

Ben-Haim (2001, 2006, p. xii)

In a nutshell, the rhetoric in the literature on info-gap decision theory would have us believe that a **simple instance** of the most famous non-probabilistic **prototype** robustness model in the trade (circa 1940) is radically different from all current robustness models, including the **prototype** itself!!

Surely, this fact justifies labeling info-gap's robustness model the **instance that roared** (Sniedovich, 2012e).

9 So what's next on the Risk Analysis/Info-Gap agenda?

Considering the *Journal's* track record on info-gap decision theory over the past 10 years[5], one wonders what to expect next. Because, if Ben-Haim (2012) is anything to go by, then it would seem that . . . the opportunities for this theory are **limitless**!

To see that this conclusion is not as far fetched as it might appear at first, take note that as the last 10 year have shown, even referees of a journal specializing in *risk analysis* prove gullible to the unsubstantiated rhetoric in the literature on info-gap decision theory about the basic properties of *Wald' maximin* and *worst-case analysis.* For surely, claims such as this

> The min-max concept responds to severe uncertainty that nonetheless can be bounded.
>
> Ben-Haim (2012, p. 1644)

have no place in a peer-reviewed journal specializing in *risk analysis*.

It is also important to point out that, as indicated in the article *Fooled by local robustness . . . again!* (Sniedovich 2012d), *Risk Analysis* referees apparently fail to distinguish between a *parametric analysis* with respect to one parameter, and *global robustness* against uncertain variations in the value of another parameter. For otherwise, how could they have possibly approved a fallacy such as this:

> For small α, searching set $U(\alpha, \tilde{u})$ resembles a local robustness analysis. However, α is allowed to increase so that in the limit the set $U(\alpha, \tilde{u})$ covers the entire parameter space and the analysis becomes one of global robustness. The analysis of a continuum of uncertainty from local to global is one of the novel ways in which info-gap analysis is informative.
>
> Hall et al. (2012, pp. 1661-1662)

More on this in Sniedovich (2012d).

That said, the question is whether there are indications that the *Journal* is beginning to see info-gap decision theory for what it is. For consider this statement in the recent article *Confronting Deep Uncertainties in Risk Analysis* by Cox (2012):

> Table I summarizes 10 tools that can help us to better understand deep uncertainty and make decisions even when correct models are unknown. They implement two main strategies: finding *robust decisions* that work acceptably well for many models (those in the uncertainty set); and *adaptive risk management,* or learning what to do by well-designed and analyzed trial and error. Each is discussed in the following paragraphs, which also explain the different columns for generating, optimizing/adapting, and combining multiple model results.
>
> Cox (2012, p. 1611)

Clearly, info-gap decision theory is not included in Cox's (2012) short list of 10 tools.

Nonetheless, this omission is rather surprising, because given the *Journal*'s record on publications dealing with info-gap decision theory[5], it is clear that no other decision theory, claiming to provide the requisite means for managing **severe** uncertainty, matches info-gap decision theory's publication record in the *Journal* over

[5]See http://www.moshe-online.com/Risk-Analysis-101/

the past 10 years or so! Indeed, this omission is doubly surprising considering that two articles on this theory (Ben-Haim 2012; Hall et al. 2012) accompany Cox's (2012) article in the same issue of the *Journal* [5].

By this criterion, Table I in Cox (2012) in fact fails to give an adequate representation of the discourse in the *Journal* over the past 10 years or so, on non-probabilistic methods for dealing with **severe** uncertainty. To put it bluntly, it is rather odd that Cox (2012) gives no indication whatsoever of the methods for dealing with severe uncertainty that have been discussed over the past 10 years or so in the wider literature on *risk analysis,* and particularly in the journal *Risk Analysis*.

Is this a sign then that, despite its record over the part 10 years in the *Journal,* info-gap decision theory is no longer considered a risk analysis tool for coping with severe uncertainty?

We shall have to wait and see.

Bibliography

- Anderson, P. and Bernfeld, S.R. (2001) Properties of the radii of stability and instability. Pp. 1-9 in *Differential Equations and Nonlinear Mechanics,* K. Vajravelu (editor), Kluwver, Norwell.
- Ben-Haim, Y. (2001) *Information-gap decision theory.* Academic Press.
- Ben-Haim, Y. (2005) Value-at-risk with info-gap uncertainty. *The Journal of Risk Finance,* 6(5), 388-403.
- Ben-Haim, Y. (2006) *Info-gap decision theory.* Elsevier.
- Ben-Haim, Y. (2007) Faqs about info-gap. http://www.technion.ac.il/yakov/IGT/faqs01.pdf
- Ben-Haim, Y. (2010) *Info-gap Economics: an Operational Introduction.* Palgrave.
- Ben-Haim, Y. (2012) Why Risk Analysis is Difficult, and Some Thoughts on How to Proceed. *Risk Analysis* 32(10), 1638-1646.
- Ben-Haim, Y., Dacso, C., Carrasco, J., and Rajan, N. (2009) Heterogeneous uncertainties in cholesterol management. *International Journal of Approximate Reasoning* 50, 1046-1065.
- Cox, L.A. (2012) Confronting deep uncertainties in risk analysis. *Risk Analysis,* 32(10), 1607-1629.
- Demyanov, V.M., and Malozemov, V.N. (1990) *Introduction to Minimax.* Dover, NY.
- Du, D-Z., and Pardalos, P.M. (1995) *Minimax and Applications.* Kluwer.
- French, S.D. (1988) *Decision Theory.* Ellis Horwood, NY.
- Hall, J., Lempert, R.J., Keller, K., Hackbarth, C., Mijere, A., and McInerney, D.J. (2012) Robust climate policies under uncertainty: A comparison of Info-Gap and RDM methods. *Risk Analysis,* 32(10), 1657-1672.
- Hindrichsen D. and Pritchard, A.J. (1986a) Stability radii of linear systems. *Systems & Control Letters,* 7, 1-10.
- Hindrichsen D. and Pritchard, A.J. (1986b) Stability radius for structured perturbations and the algebraic Riccati equation. *Systems & Control Letters,* 8, 105-113.
- Milne, W.E. and Reynolds, R.R. (1962) Fifth-order methods for the numerical solution of ordinary differential equations. *Journal of the ACM,* 9(1), 64-70.
- Rawls, J. (1971) *Theory of Justice.* Belknap Press, Cambridge, MA.
- Resnik, M.D. (1987) *Choices: an Introduction to Decision Theory.* University of Minnesota Press, Minneapolis, MN.
- Rustem, B., and Howe, M. (2002) *Algorithms for Worst-case Design and Applications to Risk Management.* Princeton University Press, Princeton.
- Schwartz, B., Ben-Haim, Y., and Dacso, C. (2011) What Makes a Good Decision? Robust Satisficing as a Normative Standard of Rational Decision Making. *The Journal for the Theory of Social Behaviour,* 41(2), 209-227.
- Sniedovich, M. (2007) The art and science of modeling decision-making under severe uncertainty. *Decision Making in Manufacturing and Services,* 1-2, 111-136.

- Sniedovich, M. (2008) Wald's Maximin model: a treasure in disguise! *Journal of Risk Finance,* 9(3), 287-291.
- Sniedovich, M. (2008a) The Mighty Maximin. Working Paper No. MS-02-08, Department of Mathematics and Statistics, The University of Melbourne. http://info-gap.moshe-online.com/mighty.pdf
- Sniedovich, M. (2010) A bird's view of Info-Gap decision theory, *Journal of Risk Finance,* 11(3), 268-283.
- Sniedovich, M. (2012) Black swans, new Nostradamuses, voodoo decision theories and the science of decision-making in the face of severe uncertainty. *International Transactions in Operational Research,* 19(1-2), 253-281.
- Sniedovich, M. (2012a) Fooled by local robustness. *Risk Analysis* 32(10), 1630-1637.
- Sniedovich, M. (2012b) Fooled by local robustness: an applied ecology perspective. *Ecological Applications* 22(5), 1421-1427.
- Sniedovich, M. (2012c) Robust-Optimization: the elephant in the robust-satisficing room. Working Paper SM-12-1, Department of Mathematics and Statistics, The University of Melbourne. http://www.moshe-online.com/Risk-Analysis-101/elephant.pdf
- Sniedovich, M. (2012d) Fooled by local robustness . . . again! Working Paper SM-12-2, Department of Mathematics and Statistics, The University of Melbourne. http://www.moshe-online.com/Risk-Analysis-101/fooled_again.pdf
- Sniedovich, M. (2012e) Rhetoric and demagoguery in risk analysis, Part I: Wald's Mighty Maximin Paradigm. Working Paper SM-12-3, Department of Mathematics and Statistics, The University of Melbourne. http://www.moshe-online.com/Risk-Analysis-101/mighty_ra.pdf
- Sniedovich, M. (2012f) Rhetoric and demagoguery in risk analysis, Part II: anatomy of a peer-reviewed voodoo decision theory. Working Paper SM-12-4, Department of Mathematics and Statistics, The University of Melbourne. http://www.moshe-online.com/Risk-Analysis-101/voodoo_ra.pdf
- Wald, A. (1939) Contributions to the theory of statistical estimation and testing hypotheses. *Annals of Mathematical Statistics* 10(4), 299-326.
- Wald, A. (1945) Statistical decision functions which minimize the maximum risk. *The Annals of Mathematics* 46(2), 265-280.
- Wald, A. (1950). *Statistical Decision Functions.* John Wiley, NY.
- Wilf, H.S. (1960) Maximally stable numerical integration. *Journal of the Society for Industrial and Applied Mathematics,* 8(3), 537-540.
- Zlobec, S. (1987) Survey of input optimization. *Optimization,* 18:309-348.
- Zlobec, S. (1988) Characterizing Optimality in Mathematical Programming Models. *Acta Applicandae Mathematicae,* 12, 113-180.
- Zlobec, S. (2001) Nondifferentiable optimization: Parametric programming. In *Encyclopedia of Optimization,* Floudas, C.A and Pardalos, P.M. editors, Springer.