Working Paper SM-12-4

Risk Analysis 101
Rhetoric in risk analysis, Part II: anatomy of a peer-reviewed voodoo decision theory*

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Last update: July 6, 2013

Preface

If the proliferation of expressions such as *voodoo economics*, *voodoo science*, *voodoo accounting*, *voodoo mathematics*, *voodoo physics*, *voodoo statistics*, *voodoo programming*, *voodoo criminology,* is anything to go by, then it would seem that voodoo theories and practices are well represented in many disciplines.

Still, even if we accept this to be an inevitable fact of the scientific (academic) endeavor, it should prove interesting to find out how *voodoo theories* manage to gain a foothold in scientific disciplines. In fact, it should prove even more interesting to uncover how such theories manage to pass muster in the peer-review process in the first place.

In an attempt to throw some light on this phenomenon, I examine in this article how a decision theory, namely *info-gap decision theory,* which clearly deserves the title *voodoo theory,* managed to obtain the sanction of the referees of peer-reviewed journals, such as the journal *Risk Analysis,* and to thereby gain a foothold in a number of disciplines concerned with risk analysis.

My thesis is that the only reason that info-gap decision theory managed to pass muster in the peer review process is that rhetoric played a vital role in making this possible. To be precise, I contend that the rhetoric that accompanies this theory misrepresent the hard facts about it to such an extent that referees of journals, such as *Risk Analysis,* are unable to make a correct assessment of what this theory is and what it actually does. This claim is based, among other things, on a number of articles on info-gap decision theory that were published recently in the journal *Risk Analysis.*

But first a historical perspective.

In the past eight years, I was surprised, time and again, by the fact that a theory such as info-gap decision theory, that by any measure ought to have been easily identified for what it is, namely a *voodoo* decision theory, has managed to fool so many senior risk analysts about its real nature, its mode of operation, its capabilities and its scope. What I find so incomprehensible is that so many risk analysts fail to see what is so obviously clear. Namely, that locally oriented radius of stability models (circa 1960), such as info-gap's robustness model (circa 2000), are indeed models of *local* robustness. And what is more, that such models are utterly unsuitable for the task that

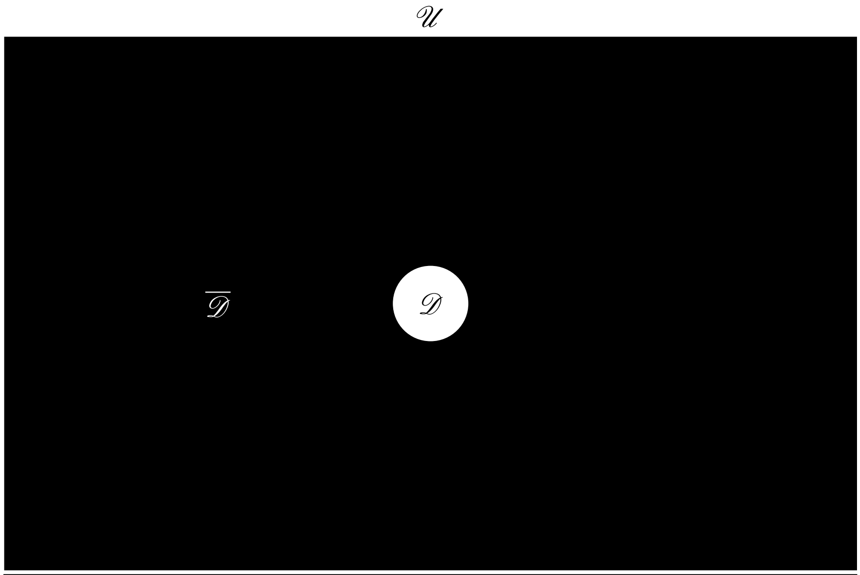
*This article was written for the **Risk Analysis 101 Project** as a Second Opinion on assessments published recently in *Risk Analysis* on decision-making under severe uncertainty. See Risk-Analysis-101.moshe-online.com.

info-gap's robustness model is claimed to have been designed for, namely seeking decisions that are robust against severe uncertainty.

The point is that to verify these facts one need not engage in an elaborate, technical, mathematical analysis. All this can be easily verified with the aid of simple *graphic* illustrations.

So before I guide you through the technical, mathematical, aspects of radius of stability models of the type used by info-gap decision theory, let me explain what I mean by the term *voodoo decision theory* in the context of this discussion and why I regard info-gap decision theory as a *voodoo* decision theory par excellence.

Consider the situation depicted in Figure 1, where the idea is to measure the performance of System B127. The large rectangle, denoted $\mathscr{U}$, represents the set of all the possible/plausible values of some parameter u, and the small white circle, denoted $\mathscr{D}$, represents the domain of the analysis under consideration. The symbol $\mathscr{U}$, in the case of info-gap decision theory, represents the *uncertainty space* of the problem concerned. I shall use the terms *uncertainty space* and *prameter space* interchangeably.



please answer the following question:

Given that an extensive performance analysis of System B127 found it to perform reasonably well on $\mathscr{D}$, how would you expect it to perform on $\mathscr{U}$?

Tick the most appropriate box.

- ☐ Well
- ☐ Reasonably well
- ☐ Exceptionally well
- ☐ Badly
- ☐ Considerably badly
- ☐ Exceptionally badly
- ☐ Other (please explain)

Figure 1: Parameter space, $\mathscr{U}$, and domain of analysis, $\mathscr{D}$, of System B127

The black area, denoted $\overline{\mathscr{D}}$, represents the complement of $\mathscr{D}$, namely $\overline{\mathscr{D}} := \mathscr{U} \setminus \mathscr{D}$. The results of the analysis take no account whatsoever of the performance of System B127 on this part of the parameter space. For obvious reasons I refer to this region of the parameter space as *No Man's Land*.

Now, suppose that based on an extensive and expensive analysis, it was determined that System B127 performs "reasonably well" on $\mathscr{D}$, namely the system performs reasonably well for values of the parameter u in set $\mathscr{D}$.

Suppose next that, as risk analysis consultants, we were hired (for a fee + expenses) to advise an important client, our brief being to answer the question posted at the bottom of Figure 1.

Before rushing to fill-in the form, we need to be clear on the issues that are raised by the question under consideration. First, the implications of the distinction between $\mathscr{U}$ and $\mathscr{D}$:

- The parameter space, $\mathscr{U}$, represents the set of all the possible/plausible values of parameter u under consideration.
- The domain of the analysis, $\mathscr{D}$, represents only those values of parameter u that are active in the analysis, that is, values that may have some impact on the analysis, and consequently on the results generated by the analysis. Values of parameter u in the *No Man's Land* $\overline{\mathscr{D}} := \mathscr{U} \setminus \mathscr{D}$ have no impact whatsoever on the results generated by the analysis. It is as though they do not exist.

Second, the special characteristics of the analysis implied by our task:

- The domain of the analysis, $\mathscr{D}$, is significantly smaller than the parameter space $\mathscr{U}$.
- The domain of the analysis, $\mathscr{D}$, consists of values of u that are in the same neighborhood of the parameter space $\mathscr{U}$.
- We are completely in the dark as to the performance of System B127 outside the domain of the analysis $\mathscr{D}$.

Need one say more?

It is particularly important to take note that $\mathscr{D}$ is a *neighborhood* in $\mathscr{U}$. That is, it is important to keep in mind that the points in $\mathscr{D}$ are not distributed over the parameter space $\mathscr{U}$. This means that even if System B127 behaves "smoothly" over $\mathscr{U}$, these points do not provide a good approximation of the variability of u over $\mathscr{U}$.

The underlying idea is that an analysis is *global* if the domain of the analysis is equal to the parameter space, namely if $\mathscr{D} = \mathscr{U}$. In contrast, an analysis is *local* if the domain of the analysis is a subset of the parameter space consisting of values of the parameters that are from the same *locale* or *neighborhood* in the parameter space $\mathscr{U}$.

The inference is clear. The analysis depicted in Figure 1 is *local* because its domain $\mathscr{D}$ consists of a small subset of the parameter space $\mathscr{U}$ all of whose elements are from the same *locale* or *neighborhood* in the parameter space.

In comparison, consider the situation shown in Figure 2, where as in Figure 1, set $\mathscr{D}$ is represented by the white area (lines) and its complement $\overline{\mathscr{D}}$ is represented by the black area. Obviously, in this case as well, set $\mathscr{D}$ is a small subset of $\mathscr{U}$, and therefore, it may not be an appropriate "approximation" of $\mathscr{U}$.

However.

In contrast to the situation depicted in Figure 1, here the white area is not concentrated in a single "neighborhood" around a point in $\mathscr{U}$. This means that the analysis in this case is not local because it is based on values of u that are distributed over $\mathscr{U}$, rather than on values that are concentrated in a single neighborhood of $\mathscr{U}$. All the same, such an analysis is not global either.

The inference is that the two decisive features determining that an analysis is local are these:

- The domain of the analysis, $\mathscr{D}$, is a relatively *small* subset of the parameter space $\mathscr{U}$.
- The domain of the analysis, $\mathscr{D}$, is a single *neighborhood* of the parameter space $\mathscr{U}$.

Combined, these two features yield the following broad definition of a *local analysis*:

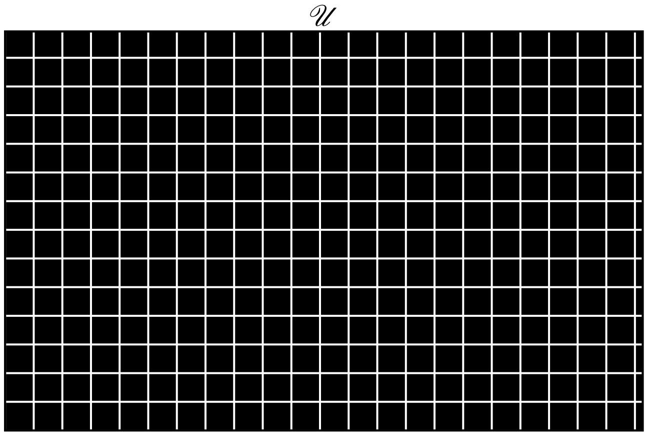


Figure 2: Parameter space $\mathscr{U}$ and domain of analysis (white lines) of System B128

> An analysis is *local* if its domain $\mathscr{D}$ is a relatively small neighborhood of the parameter space $\mathscr{U}$ under consideration.

How then does the epithet *voodoo decision theory* enter the picture?

In this discussion the term *voodoo decision theory* refers to a decision theory, such as info-gap decision theory, that claims to offer a reliable tool for robust decision-making in the face of an extreme uncertainty, when the means that it provides for this purpose turns out to be a local robustness analysis.

It is important to keep in mind that the term *extreme* uncertainty has a precise meaning here such that *extreme* is manifested in these three properties:

- The uncertainty space $\mathscr{U}$ is vast. It can be unbounded.
- The point estimate of the true value of the parameter of interest is a poor guess. It can even be a wild guess.
- The quantification of the uncertainty is non-probabilistic, likelihood-free, belief-free, etc.

These properties also characterize the *severe* uncertainty postulated by info-gap decision theory.

My thesis is then that rhetoric hampers a correct assessment of info-gap decision theory by obscuring from view the profound incongruity between what this theory claims to be and what it really is. Thus, while the rhetoric proclaims it to be a reliable tool for the treatment of situations where the parameter space, $\mathscr{U}$, is typically **unbounded,** the hard facts are that the domain of its analysis, $\mathscr{D}$, is a relatively **small neighborhood** around a point estimate $\tilde{u}$ that is no more than a **guess** of the true value of the parameter of interest. This is illustrated in Figure 3.

Because it is important to make clear where referees of journals, such as *Risk Analysis,* went wrong, I show/proof formally, and I illustrate graphically, that info-gap's robustness model **does not seek** decisions that are robust against the variability of u over the uncertainty space $\mathscr{U}$, **but rather** decisions that are robust against small perturbations in a nominal value of the uncertainty parameter.

I particularly call the attention of past and future referees to the fact that, as proclaimed by the info-gap literature, "the most commonly encountered info-gap models are unbounded" (Ben-Haim 2001, 2006). This means that the characterization of info-gap's prescription for the pursuit of robustness to severe uncertainty, as a voodoo prescription *par excellence,* holds for typical applications of info-gap decision theory.

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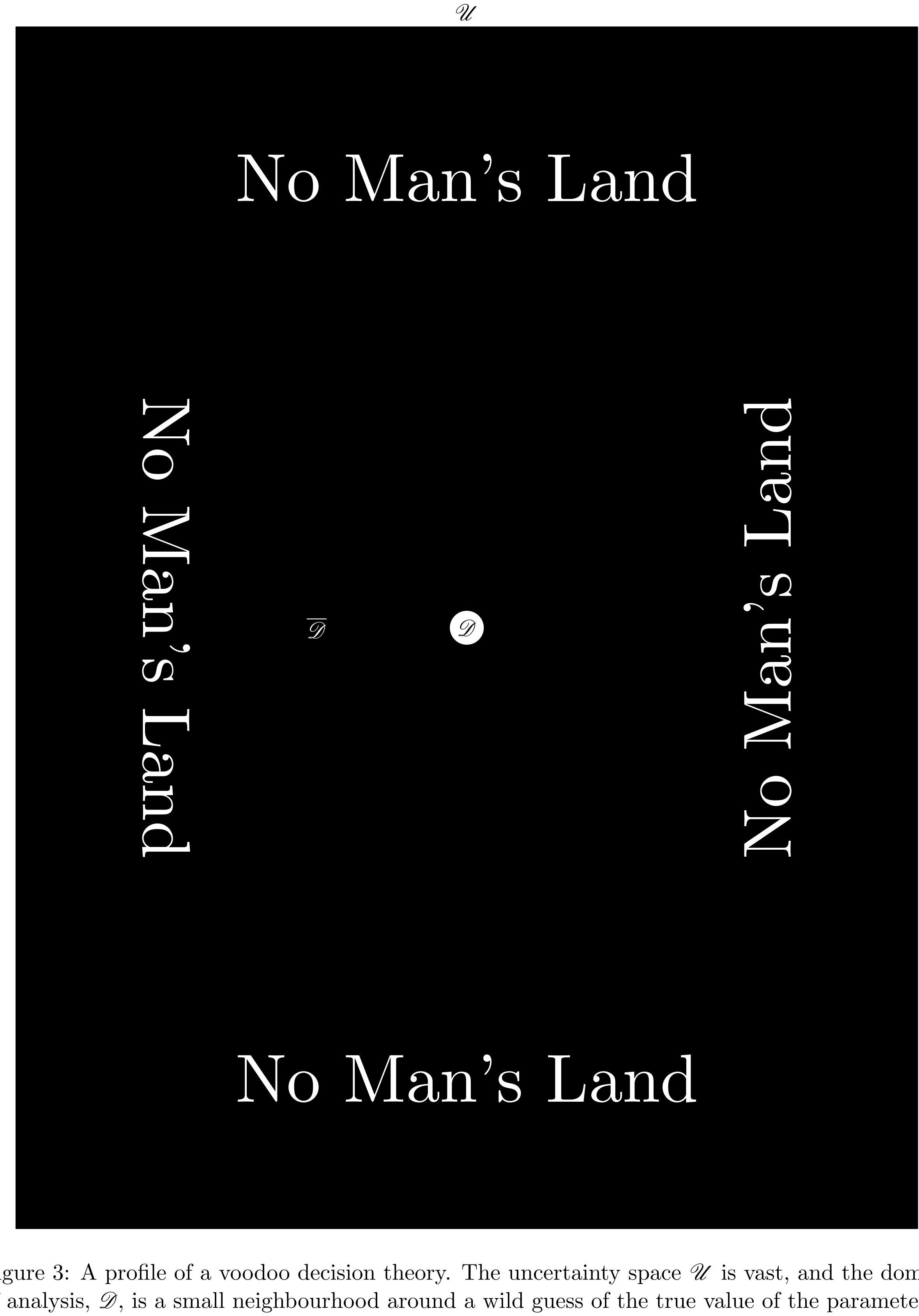

Figure 3: A profile of a voodoo decision theory. The uncertainty space $\mathscr{U}$ is vast, and the domain of analysis, $\mathscr{D}$, is a small neighbourhood around a wild guess of the true value of the parameter of interest.

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1 Introduction

Consider the following highly confidential decision-making situation involving a vast parameter space, denoted $\mathscr{U}$, and a relatively small neighborhood in it, denoted $\mathscr{D}$, representing the domain of a secret analysis that was conducted to determine the performance of SYSTEM Z34 on $\mathscr{U}$. We know practically nothing about this secret analysis. But, we do know that the parameter space $\mathscr{U}$ represents the mainland of Australia, and $\mathscr{D}$ represents *Melbourne's metropolitan area.* Some non-confidential information about Australia and Melbourne is provided in Figure 4.

About Australia: Australia, officially the Commonwealth of Australia, is a country in the Southern Hemisphere comprising the mainland of the Australian continent as well as the island of Tasmania and numerous smaller islands in the Indian and Pacific Oceans. In land area, Australia is estimated to be 7,692,024 square kilometers and the sixth largest nation after Russia, Canada, China, the United States of America and Brazil. More than 58 million kangaroos, 50,000 koalas, and 150,000 saltwater crocodiles reside in this vast land. However, the human population is relatively small—about 22 million—and is concentrated in seven cities.

About Melbourne: Melbourne was founded in 1835 (47 years after the European settlement of Australia). It is the capital and most populous city in the state of Victoria, and the second most populous city in Australia. The Melbourne City Centre is the hub of the greater metropolitan area and the Census statistical division—of which "Melbourne" is the common name. As of June 2010, the greater geographical area had an approximate population of four million. Inhabitants of Melbourne are called Melburnians or Melbournians.

Figure 4: About Australia and Melbourne

The point I want to make straightaway is that just as one does not have to be an anesthesiologist to grasp the implication of the concept local anesthesia, one does not have to be a risk analyst to appreciate what a *local analysis* is.

The point is that the fact that domain $\mathscr{D}$ is *local* does not require the precise specification of its dimension in order to make it abundantly clear that one deals with a *local* domain. This can be determined exactly, or approximately, or roughly, in the context of the problem under consideration. Thus, in Figure 4, the secret analysis conducted in Melbourne's metropolitan area is *local* because, irrespective of the exact boundaries of Melbourne's metropolitan area, hence the exact or approximate "size" of $\mathscr{D}$ representing this area, relative to the parametric space *Australia,* an analysis conducted in this domain is *local.*

I need hardly point out that the concept *local analysis* is indispensable both as a tool of thought and as a practical device. It is essential therefore to take full note of what this concept allows and what it disallows. It is particularly important to appreciate the limitations of a *local analysis* so as to avoid using it in situations where a *global* analysis is required. Failure to appreciate this point can lead to gross misapplications of this invaluable concept.

A vivid example of such a gross misapplication is info-gap decision theory where, a misapprehension of the fundamental difference between *local analysis* and *global analysis* leads to a total blurring of the two types of analysis. In fact, the trouble in info-gap runs deeper than that. The real problem in info-gap is a more basic lack of appreciation that such a distinction exists. That this is indeed so is evidenced by info-gap's primary texts (Ben-Haim 2001, 2006, 2010) where, not only is this distinction not discussed, it is not even noted. Add to this the fact that the rhetoric in this literature gives a totally misleading account of how this theory works, and what it is actually capable of, and it is clear why so many info-gap followers have no idea that info-gap's robustness model is in fact inherently local.

Indeed, some info-gap followers labour under the misconception that info-gap robustness is a measure of global robustness. Better yet, some info-gap adherents even go so far as to declare that info-gap decision theory is innovative in that it allows controlling the scope of the robustness analysis from *local* to *global* (Hall et. al. 2012).

Now.

As I indicated already, I label info-gap decision theory a *voodoo* decision theory *par excellence* because of the huge incongruity between what this theory claims to be doing and what it actually does. Thus, while it claims to provide a reliable tool for the treatment of situations where the most commonly encountered parameter space, $\mathscr{U}$, is **unbounded,** the model it puts forward for this purpose is a *local* robustness model *par excellence.* This means that what this model **in fact** prescribes doing is to focus the analysis on the domain $\mathscr{D}$, that is a minutely (infinitesimally) small neighborhood of the parametric space $\mathscr{U}$. In other words, for all the fuss about the parameter space, $\mathscr{U}$ being vast, indeed **unbounded,** when it comes to the analysis prescribed by this model, the domain of the analysis, $\mathscr{D}$, is a neighborhood around an element $\tilde{u}$ that represent a "wild guess" of the true value of the parameter $u \in \mathscr{U}$. This is illustrated schematically in Figure 5.

For a more concrete idea of what this means, take another look at Figure 3. Assume that $\mathscr{U} = \mathbb{R}^2$, where $\mathbb{R}$ denotes the real line, and $\mathscr{D}$ is a circle of radius α, for some finite α. Can there be any doubt that the analysis in this case ignores most of the possible/plausible values of the parameter $u \in \mathscr{U}$?

On what grounds, then, can it possibly be claimed that an analysis conducted on the small circle $\mathscr{D}$ is particularly suitable for the treatment of a severe uncertainty manifested in *rare events, extreme events, shocks, surprises, catastrophes, tsunamis,* and so on, (Ben-Haim 2010) that are represented by values of u that can be very distant from $\mathscr{D}$?

Since such claims are not only groundless but in fact absurd, the only epithet befitting a theory propounding such claims is *voodoo theory par excellence.*

The trouble is of course that in the case of info-gap decision theory, the hard facts are fogged up by a misleading rhetoric. Indeed, it is only when this rhetoric is peeled away that one discovers that the **reliable** recipe that this theory prescribes for the treatment of severe uncertainty comes down to this:

> A reliable $1 \diamond 2 \diamond 3$ recipe for the treatment of severe uncertainty
>
> 1. **Ignore** the severity of the uncertainty.
> 2. Select a **guess** of the true value of the parameter of interest.
> 3. Conduct a **local** robustness analysis in the **neighborhood** of this **guess.**

Having said all that, I want to make my characterization of info-gap decision theory as a voodoo decision theory abundantly clear.

By exposing the absurd in info-gap's prescription for the management of severe uncertainty I do not mean to categorically deny that there can be situations where a local analysis in the neighborhood of a "wild guess" can be a reliable tool for the treatment of a case subject to a severe uncertainty of the type addressed by info-gap decision theory. Of course, such situations may occur (see Sniedovich 2011).

The point is, however, that this obviously is not the rule. And the implication is that the burden of proof is on info-gap decision theory. In other words, so long as info-gap decision theory does not provide a rigorous justification for its claims that its methodology offers a reliable tool for the management of a severe uncertainty of the type that it stipulates, there are no grounds to accept them. Indeed, there is every ground to assert that such claims border on the absurd, because it is elementary to construct counter example showing that such claims are groundless if not farcical. This can be easily corroborated analytically and illustrated graphically.

To put a different spin on this matter, it is important to point out the following. The fact that under certain conditions the local robustness yielded by info-gap's robustness analysis theory is

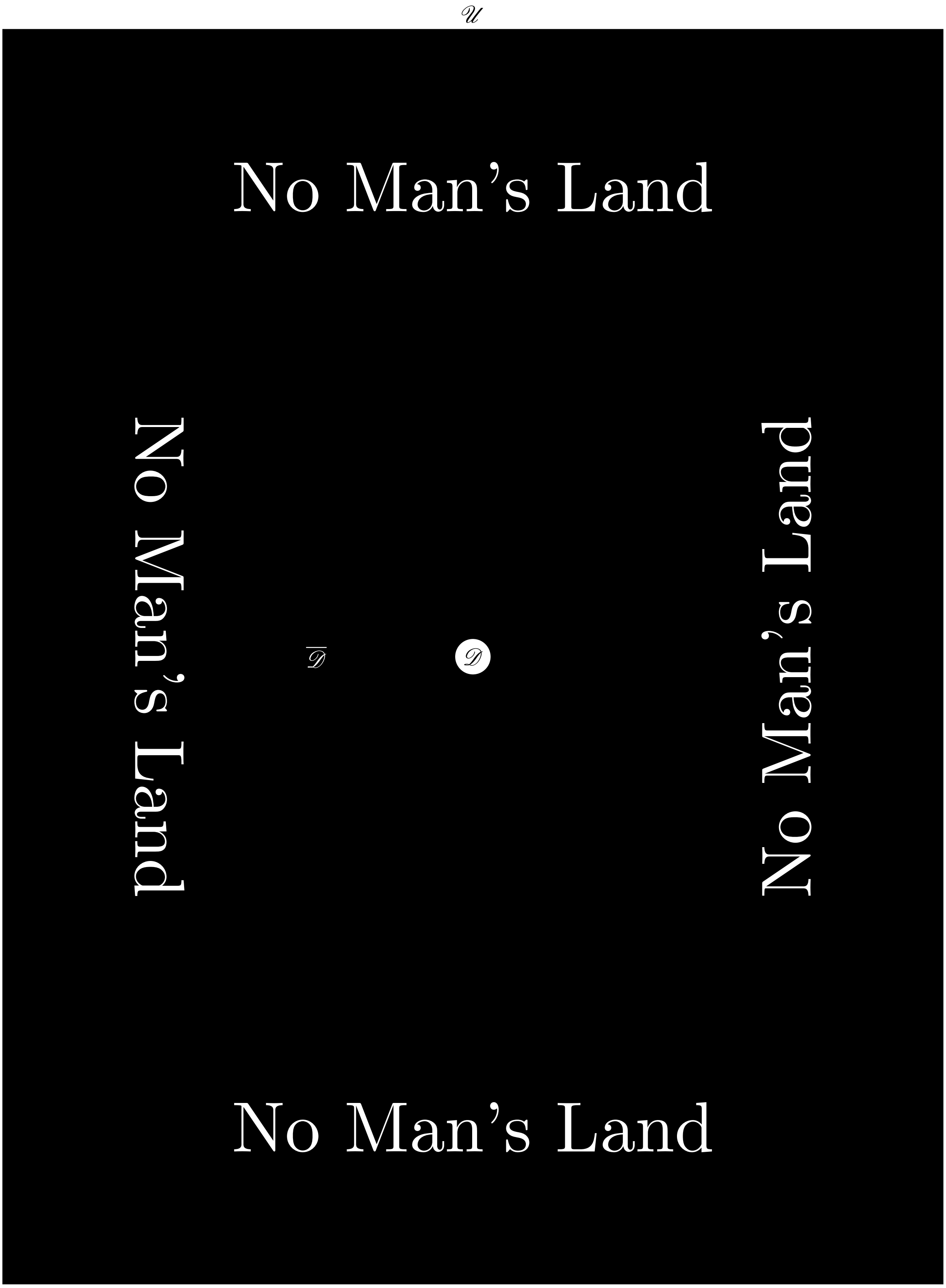

Figure 5: A profile of info-gap's robustness analysis. The uncertainty space $\mathscr{U}$ is vast, and the domain of analysis, $\mathscr{D}$, is a small neighborhood around a wild guess of the true value of the parameter of interest.

a valid approximation of global robustness, measured over the uncertainty space, cannot absolve info-gap's methodology from the charge that it is a voodoo methodology par excellence. The point here is that such a result **speaks not** for the methodology *per se* but for the specific case and conditions that yielded it.

The closest analogy one can draw is with *optimization theory,* specifically with the distinction between *local* and *global* optimization. The fact that under certain conditions a *local optimum* is also a *global optimum* does not imply that the specific local optimization method that yielded this result is a reliable tool for generating global optimal solutions. And by the same token, the fact that a dead watch shows the correct time twice a day, does not imply that this watch is a reliable tool for determining the time of day over a period of, say 2 years.

To determine the scope of a methodology, namely to determine whether it is based on **local** or **global** robustness, the question that we should ask ourself is this: which of the following two questions does its robustness model address?

- **Global robustness question:**
  How robust is a decision against variations in the value of the uncertainty parameter over its assumed set of possible/plausible values?
- **Local robustness question:**
  How robust is a decision against small perturbations/deviations in a nominal value of the uncertainty parameter?

Only a methodology that addresses the first question is based on global robustness. Hence, for all the rhetoric in the info-gap literature, which would have us believe that info-gap decision theory furnishes a methodology of global robustness, the fact of the matter is that, by definition/construction, info-gap's robustness model addresses the second question. It is therefore a methodology of local robustness *par excellence.* So, if you appreciate the fundamental difference between these two questions, you appreciate why this theory deserves the title voodoo decision theory *par excellence.*

Incidentally, the term *voodoo* in *voodoo decision theory* functions in precisely the same manner as in *voodoo economics*, *voodoo science*, *voodoo mathematics*, *voodoo statistics*, and so on. So roughly, in this discussion a voodoo decision theory is a theory that lacks sufficient evidence or proof and/or is based on flawed logic, utterly unrealistic and/or contradictory assumptions, spurious correlations, and so on.

I should point out, though, that the term *Voodoo Decision Theory* is not my coinage (what a pity!):

> The behavior of Kropotkin's cooperators is something like that of decision makers using the Jeffrey expected utility model in the Max and Moritz situation. Are ground squirrels and vampires using voodoo decision theory?
>
> Brian Skyrms (1996, p. 51)
> *Evolution of the Social Contract*
> Cambridge University Press.

Having said all that, it should still prove instructive—especially to those readers who may not be sufficiently versed in the areas of expertise on which info-gap decision theory relies—to have on hand a benchmark *voodoo decision theory* against which they would be able to judge for themselves whether info-gap decision theory is indeed a *voodoo* decision theory.

The trademark of such a benchmark theory would be the unsubstantiated groundless claim that the theory provides a reliable methodology for the treatment of a decision problem that is subject to an *extreme* uncertainty. This gives a general outline of such a theory's approach. To be able to give it content, it is necessary to be clear on the specifics of what such a theory means by an *extreme* uncertainty and, the type of decision problem that it is concerned with.

2 What is the problem?

To avoid cluttering the discussion with nonessential details, all we need to be clear on is the generic problem that such a theory addresses by defining the objects/constructs comprising it. Obviously, in each case the user gives a determinate specification to these objects, based on the information available about the decision-making situation under consideration. Thus:

$$\begin{aligned}
Q &= \text{set of available } \textit{decisions.}\\
\mathscr{U} &= \textit{ uncertainty space} \text{ of parameter } u.\\
\tilde{u} &= \textit{point estimate} \text{ of the true value of the uncertainty parameter } u.\\
A(q) &= \text{set of } \textit{acceptable values} \text{ of } u \text{ associated with decision } q \in Q.
\end{aligned}$$

The sets $A(q), q \in Q$ are typically specified as *performance constraints* imposed on the (q, u) pairs, namely constraints that decision q is required to satisfy. Thus, in terms of $A(q)$, the constraints imposed on q can be expressed as:

$$u \in A(q). \tag{1}$$

For example, if decision q is required to satisfy the constraints $0 \leq r(q, u)$ and $t(q, u) \in T(q)$, then we would set

$$A(q) = \{u \in \mathscr{U} : 0 \leq r(q, u), t(q, u) \in T(q)\} \ , \ q \in Q, \tag{2}$$

observing that this would imply that $u \in A(q)$ iff $0 \leq r(q, u)$ and $t(q, u) \in T(q)$.

And in the case of the generic info-gap decision model (Ben-Haim 2001, 2006), we would set

$$A(q) = \{u \in \mathscr{U} : r_c \leq r(q, u)\} \ , \ q \in Q. \tag{3}$$

The robustness issue arises because typically there is no decision $q \in Q$ such that $A(q) = \mathscr{U}$. In other words, typically $A(q)$ is only a (proper) subset of $\mathscr{U}$, meaning that decision q satisfies these constraints for some, but not all, the values of u in $\mathscr{U}$. The decision maker therefore seeks a decision whose set of acceptable values of u is large: the larger the better.

As for the uncertainty under consideration being labeled *extreme*, it is important to realize that the term *extreme* is not used loosely in this discussion. Rather, it has a determinate meaning as it must be able to give meaning to the great uncertainty surrounding the true value of u. Take note then that *extreme*, uncertainty is understood to be manifested in the following three properties:

Assumption 2.1 *Extreme Uncertainty.*
(i) The uncertainty space $\mathscr{U}$ is vast, it can be unbounded.
(ii) The point estimate $\tilde{u}$ is a poor, sometimes a wild guess of the true value of u.
(iii) The quantification of the uncertainty is non-probabilistic, likelihood-free, belief-free, etc.

An uncertainty of this nature is indeed *extreme*!

Recall that info-gap decision theory terms this type of uncertainty *severe,* rather than *extreme.* I shall therefore use the terms *severe uncertainty* and *extreme uncertainty* interchangeably.

If we bring together the elements (discussed above) that go into the design of the proposed benchmark theory, it is clear that the question addressed by it would be the following:

The Design Question:

- Given the extreme nature of the uncertainty under consideration, what measure of robustness is suitable for assessing the robustness of decision q with respect to the performance requirement $u \in A(q)$ against the variations in the value of u over $\mathscr{U}$?

This, no doubt, is an extremely challenging question. The challenge is in the onerous task presented by it.

The Challenging Task:

- Given the extreme nature of the uncertainty under consideration, formulate a suitable measure of robustness for assessing the robustness of decision q with respect to the performance requirement $u \in A(q)$ against the variations in the value of u over $\mathscr{U}$.

How this difficult task would be approached would very much depend on one's attitude to extreme uncertainty and one's understanding of the concept *robustness*.

Consider then the simple measure of global robustness discussed in the next section.

3 A benchmark measure of global robustness

The seemingly intuitive measure of global robustness outlined in this section dates back to the 1960s—the early days of *robust optimization*. I shall refer to it here as *Size-Robustness*.

It is based on the observation that in the absence of a measure of *likelihood* to "locate" the true value of u in $\mathscr{U}$, and in view of the poor quality of the point estimate $\tilde{u}$, it makes sense to measure the robustness of decision q by measuring the size of set $A(q)$ such that the larger $A(q)$ the more robust decision q.

Recall that $A(q)$ denotes the set of acceptable values of u associated with decision q, hence *Size-Robustness* measures the robustness of decision q by measuring the size of the set of acceptable values of u associated with decision q. According to this measure, the ideal decision is such that $A(q) = \mathscr{U}$, hence $size(A(q)) = size(\mathscr{U})$, where $size(V)$ denotes the size of set V, according to some suitable measure of "size". For instance, if set V consists of finitely many elements, we can let $size(V) = |V|$, where $|V|$ denotes the *cardinality* of set V.

Formally, $size$ can be regarded as a real-valued function on the power set of $\mathscr{U}$ such that $size(V) = 0$ iff V is the empty set, $size(V) > 0$ for all non-empty subsets of $\mathscr{U}$, and $V' \subset V''$ implies $size(V') \leq size(V'')$.

Definition 3.1 *Global Size-Robustness.*
The global size-robustness of decision $q \in Q$ is equal to

$$GSR(q) := size(A(q)) \ , \ q \in Q \tag{4}$$

$$= \max_{V \subseteq \mathscr{U}} \{size(V) : V \subseteq A(q)\} \tag{5}$$

$$= \max_{V \subseteq \mathscr{U}} \{size(V) : u \in A(q), \forall u \in V\}. \tag{6}$$

The larger $GSR(q)$, the more robust decision q.

Take note that it might be more effective to "normalize" this measure and use the ratio $size(A(q))/size(\mathscr{U})$, rather than $size(A(q))$, as the robustness of decision q. Alternatively, we can scale $size$ so that $size(\mathscr{U}) = 1$, in which case $0 \leq GSR(q) \leq 1$.

This is illustrated in Figure 6, where the sets of acceptable values of u associated with two decisions are represented by the shaded areas. If we measure the size of these sets by their respective "areas", then clearly we would regard decision q' to be much more robust than decision q''.

In the next section I contrast this measure of global robustness with a simple voodoo approach, called *Viva!*[1], that enables the formulation a number of benchmark voodoo decision theories.

4 Meet Viva!

The great appeal of a voodoo approach to *extreme* uncertainty is in the license it grants the designer of a prospective voodoo decision theory to feel utterly undaunted by the enormous difficulties presented by the *extreme* uncertainty under consideration.

[1]Short for *Viva la Voodoo!*

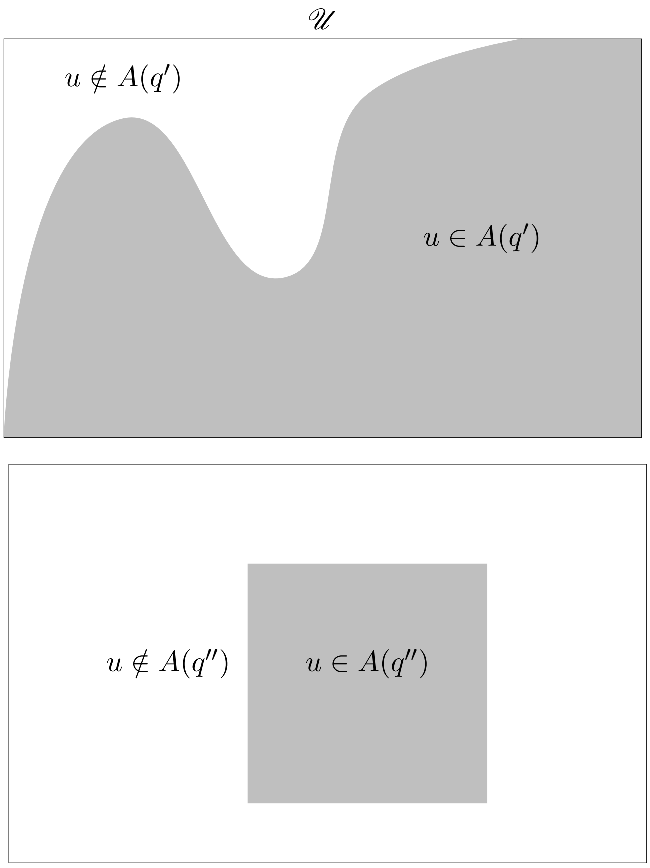

Figure 6: Size-robustness of two decisions: $size(A(q')) \gg size(A(q''))$

A designer of a prospective voodoo decision theory is thus set free of the constraints of logic, common sense, intuition, scientific conventions, etc., thus enabling him/her to adopt the simple recipe outlined in Figure 7.

Viva! $1 \diamond 2 \diamond 3$ **Recipe**

1. Ignore the fact that the uncertainty in the true value u is extreme.
2. Focus on the wild guess $\tilde{u}$.
3. Conduct a local robustness analysis in the neighborhood of $\tilde{u}$.

Figure 7: A recipe for determining the robustness of decision q

Readers who have not yet caught my drift might complain of the apparent contradiction between the *Viva!* $1 \diamond 2 \diamond 3$ Recipe and the fact that the prospective benchmark theory was supposed to be designed specifically for handling *extreme* uncertainty. Others may wonder about the idea that a very poor point estimate (guess) is at the center of a robustness analysis that is supposed to deal with a non-probabilistic, likelihood-free uncertainty.

To these readers I say that this is precisely the objective of this exercise. To demonstrate how profoundly self-contradictory a voodoo decision theory would be, and how deep the gulf between rhetoric and facts in such a theory is. Thus, to show that it is only on the back of rhetoric that such a theory would be able to make its way into journals such as *Risk Analysis*.

Neighborhoods

As indicated above, the concept *neighborhood* plays a central role in a theory espousing the *Viva!* $1 \diamond 2 \diamond 3$ **Recipe**. It is therefore important that we clarify how this concept should be construed in the context of this discussion. Let then,

$$U(\rho,\tilde{u}) := \text{neighborhood of size } \rho \text{ around } \tilde{u} \tag{7}$$
$$= \text{set of all } u \in \mathscr{U} \text{ that are within a distance } \rho \text{ (or less) from } \tilde{u} \tag{8}$$

where the distance between points in $\mathscr{U}$ is determine by a suitable measure of distance (metric/norm) on $\mathscr{U}$.

Think about the neighborhood $U(\rho,\tilde{u})$ as a "circle", or "ball," or a "box", of size $\rho \geq 0$ centered at $\tilde{u}$. We assume that

$$U(0,\tilde{u}) = \{\tilde{u}\} \quad \text{(contraction)} \tag{9}$$
$$U(\infty,\tilde{u}) = \mathscr{U} \quad \text{(expansion)} \tag{10}$$
$$\rho' < \rho'' \rightarrow U(\rho',\tilde{u}) \subseteq U(\rho'',\tilde{u}) \quad \text{(nesting)} \tag{11}$$

where $U(\infty,\tilde{u})$ denotes the limit of $U(\rho,\tilde{u})$ as $\rho \rightarrow \infty$.

What all voodoo theories espousing the *Viva!* 1⋄2⋄3 **Recipe** would have in common is that the domain of analysis, $\mathscr{D}$, would be some neighborhood $U(\rho,\tilde{u})$, where the size of the neighborhood, namely ρ, depends on the specific theory.

Behind the recipe

The rationale behind the *Viva!* 1 ⋄ 2 ⋄ 3 **Recipe** can be summed up by the following innovative, revolutionary, indeed, groundbreaking ideas:

- Keep it simple, mate!
- Go local!
- Reinvent the wheel!

Let me elaborate.

Keep it simple, mate!

As I noted above, the forbidding nature of the *extreme* uncertainty that we are dealing with here poses huge methodological and technical challenges. But these challenges simply disappear if one simply takes no account of the fact that the uncertainty under consideration is *extreme!!!*

Naive readers may no doubt object that this amounts to self-deception. And what is more, they would seriously question the practicality of marketing such a recipe. They would no doubt ask: how can one possibly "sell" a theory that is claimed to deal with *extreme* uncertainty if the theory in fact ignores the *extreme* nature of the uncertainty?

To these readers I say: never underestimate the force and effectiveness of rhetoric as marketing tools.

By this I mean that the *Keep it simple, mate!* principle adopted by the *Viva!* 1 ⋄ 2 ⋄ 3 **Recipe**, which enables to ignore the extreme nature of the uncertainty under consideration, is obviously not marketed as such. In other words, a voodoo theory would not claim in so many words that this is precisely what its "reliable method" comes down to. To the contrary, the whole point of a voodoo theory is that the rhetoric accompanying it succeed in concealing such hard facts from some risk analysts, including referees of peer-reviewed journals[2].

Go local!

The *Keep it simple, mate!* principle, which enables ignoring the fact that the uncertainty under consideration is extreme, obviously opens new avenues of approach for the management of extreme uncertainty. It thus allows (more accurately suggests to) the designer of the proposed benchmark

[2]This is not done deliberately, though. It is often the result of one falling in love with one's model/method/theory.

theory, to opt for a local robustness. In other words, if one has *carte blanche* to ignore the complexities posed by an extreme uncertainty of the type stipulated here, the most obvious thing to do next is to go for a *local* approach to robustness.

The idea is very simple. Instead of having to worry about the *extreme* variations in the value of u over the vast $\mathscr{U}$, one simply focuses on the variations in the value of u over some *neighborhood* around some given point in $\mathscr{U}$. In particular, one focuses on the variations in the value of u over some *neighborhood* $U(\rho, \tilde{u})$ around the guess $\tilde{u}$.

And if hard pressed about this move, a benchmark voodoo theory would reserve the right to refer to $\tilde{u}$ by seemingly "legitimizing" terms, such as "educated guess" or "best guess", or perhaps "best estimate", and so on. Obviously, no tests would be proposed to check the quality of this "guess". The important point is then that $\tilde{u}$ would be allowed to be a poor estimate of the true value of u, with the potential of being substantially wrong.

So, what all this comes down to is that the domain of the analysis would be $\mathscr{D} = U(\rho, \tilde{u})$ for some $\alpha \geq 0$ whose value may depend on the specific local robustness analysis used. Also, as indicated in the Preface, the domain $\mathscr{D}$ of a *local* analysis is a relatively small neighborhood around $\tilde{u}$ in the uncertainty space $\mathscr{U}$. The implication is therefore that such a robustness analysis would be heavily biased in favor of values of u that are in the proximity of $\tilde{u}$, hence extremely biased against values of u that are outside the domain $\mathscr{D} = U(\rho, \tilde{u})$.

In sum, such a theory would be utterly unconcerned that by allowing the point estimate $\tilde{u}$ to be a *poor guess*, even a *wild guess*, the results of its analysis might contravene the following universally accepted maxims:

- Garbage In — Garbage Out (GIGO)
- Results of an analysis are only as good as the estimates on which they are based.

Obviously, all this would be covered up by rhetoric and spin intermingled with Greek and mathematical symbols.

Reinvent the wheel!

The "Keep it simple, mate!" principle and the "Go local!" principle would conveniently take such an approach out of the inhospitable realm of *extreme uncertainty* into the tame, and far friendlier realm of *local robustness* in the neighborhood of a given point $\tilde{u}$ in $\mathscr{U}$. The question then is how would such a theory go about defining the (local) robustness of decision q in the neighborhood of the point estimate $\tilde{u}$?

The designer of such a theory might of course survey the literature to find out which models of local robustness would best suit the theory's needs, in which case one would discover the simple *worst-case analysis* over some small neighborhood around $\tilde{u}$ (circa 1940), or a *radius of stability model* centered at $\tilde{u}$ (circa 1960). But, as might be expected, the most obvious move would be to reinvent the wheel. That is, to adopt these models to measure the (local) robustness of decision q, but to proclaim them as new, and revolutionary. Namely, to refer to these models as exponents of a radically different approach to the treatment of *extreme uncertainty.*

Again, rhetoric and spin would play a vital role in putting this across.

5 Size does matter!

I want to interrupt the discussion at this stage to comment on the meaning and ramifications of a theory allowing its uncertainty space, $\mathscr{U}$, to be vast, even *unbounded.*

Ostensibly, the reason for this is to enable it to deal not only with "normal" events that would be represented by values u that presumably are near $\tilde{u}$. But also with extreme events, rare events, catastrophes, shocks, tsunamis, economic meltdowns (Ben-Haim 2010), that would be represented by values u that presumably would be at a great distance from the point estimate $\tilde{u}$. Clearly, the ability to contain both types of values often translates into a vast uncertainty space.

But, it is important to keep in mind the consequences of a *local analysis* in a vast, indeed unbounded uncertainty space. It is important to take note of the discrepancy—yielded by such an analysis—between the size of the uncertainty space $\mathscr{U}$ and the size of the domain of the analysis, namely the neighborhood $\mathscr{D} = U(\rho, \tilde{u})$ around $\tilde{u}$ on which the robustness analysis is conducted.

Naturally, such a discrepancy would be a characteristic hallmark of a benchmark voodoo theory. This is illustrated in Figure 8 where $\mathscr{D}(q)$ denotes the domain of the analysis pertaining to decision q. The levels of the yielded discrepancy translate into shades of voodoo decision theories:

Shades of voodooism	size of $\mathscr{D}(q)$ relative to the size of $\mathscr{U}$
Plain	50% − 60%
Crystal	40% − 50%
Silver	30% − 40%
Platinum	20% − 30%
Gold	10% − 20%
Par excellence	$<$ 10%

Figure 8: Semi-official shades of voodooism in decision theory

This table should be kept in mind during the discussion on the benchmark voodoo decision theories espousing the *Viva!* $1 \diamond 2 \diamond 3$ **Recipe**.

6 DIY voodoo decision theories

Recall that the third step in the *Viva!* $1 \diamond 2 \diamond 3$ **Recipe** prescribes conducting a local robustness analysis in the neighborhood of the guess $\tilde{u}$. However, since this instruction does not stipulate what robustness analysis should be implemented for this purpose, the door is left wide open for the development of a variety of different theories depending on the local robustness analysis that would be conducted. Each such theory would be based on a distinct model of local robustness. In this section I examine four such models, hence four benchmark voodoo decision theories associated with the *Viva!* $1 \diamond 2 \diamond 3$ **Recipe**.

6.1 The Mother of all Viva! theories

This theory is the most radical, because the local robustness analysis it implements is conducted on the neighborhood of $\tilde{u}$ with the latter consisting of only one element, namely $\tilde{u}$. In other words, the domain of analysis is the neighborhood $U(0, \tilde{u})$ which consists of the singleton $\{\tilde{u}\}$.

This theory can be described as follows:

Mother of all Viva! theories

1. Ignore the fact that the uncertainty in the true value u is extreme.
2. Focus on the wild guess $\tilde{u}$.
3. Rank decisions according to their performance at $\tilde{u}$.

It is hard to imagine a more representative voodoo decision theory based on the *Viva!* $1 \diamond 2 \diamond 3$ **Recipe**!

Not only that it ignores the extreme uncertainty in the true value of u, it is utterly oblivious to the question of robustness. Because, by ranking decisions solely according to their performance at $\tilde{u}$, it does not address the question of the decisions' ability to cope with the variability of u.

And, to top it all off, according to this theory, there are only two types of decisions, namely decision $q \in Q$ is either *acceptable* or *unacceptable* (at $\tilde{u}$) depending on whether $\tilde{u} \in A(q)$ or $\tilde{u} \notin A(q)$, respectively. Obviously, acceptable decisions are better than unacceptable decisions.

The implication is therefore that this theory is unable to discriminate properly between decisions, because in typical applications, numerous decisions may turn out to be acceptable. So a secondary criterion may have to be used to narrow the field[3].

6.2 Local maximin theory for a pre-determined neighborhood

A more moderate version than the *Mother of all Viva! theories,* is one that is based on Wald's maximin paradigm. Here decisions are ranked according to their worst outcome over a small neighborhood $U(\rho, \tilde{u})$ around $\tilde{u}$ whose size ρ is specified in advance. Let ρ^* denote this fixed value of ρ.

In line with Wald's maximin paradigm, this theory would then select the decision(s) whose worst outcome over the neighborhood $U(\rho^*, \tilde{u})$ is best.

Observe that in the framework of this theory there are at most two possible *outcomes* for decision q with respect to the values of u in the neighborhood $U(\rho^*, \tilde{u})$. That is, either $u \in A(q)$ or $u \notin A(q)$. Hence, a worst outcome always exists, so the existence of a worst outcome is not an issue. More specifically, let the outcome associated with the pair (q, u) be defined as follows:

$$O(q,u) := \begin{cases} 1 & , \quad u \in A(q) \\ 0 & , \quad u \notin A(q) \end{cases} \ , \ q \in Q, u \in \mathscr{U}. \tag{12}$$

Interpret the 1 as an indication that the outcome is "acceptable" and the 0 as an indication that the outcome is "unacceptable".

Obviously, the larger the outcome, the better, hence the worst outcome associated with decision q over the neighborhood $U(\rho^*, \tilde{u})$ is as follows:

$$wo(q) := \min_{u \in U(\rho^*, \tilde{u})} O(q,u) \ , \ q \in Q. \tag{13}$$

Any optimal solution u^* to this optimization problem is a worst-case of u over $U(\rho^*, \tilde{u})$ with respect to decision q. If $wo(q) = 1$, then all the elements of $U(\rho^*, \tilde{u})$ satisfy the constraint $u \in A(q)$, whereas if $wo(q) = 0$, at least one $u \in U(\rho^*, \tilde{u})$ violates this constraint.

And so, according to this theory, the best worst outcome is as follows:

$$bwo^* : = \max_{q \in Q} \ wo(q) \tag{14}$$

$$= \max_{q \in Q} \ \min_{u \in U(\rho^*, \tilde{u})} O(q,u). \tag{15}$$

If $bwo^* = 1$, then any decision q that is an optimal solution to this maximin model satisfies the constraint $u \in A(q)$ for all $u \in U(\rho^*, \tilde{u})$. If $bwo^* = 0$, then there is no decision $q \in Q$ such that $u \in A(q)$ for all $u \in U(\rho^*, \tilde{u})$.

In short,

A local maximin theory for a pre-determined neighborhood $U(\rho^*, \tilde{u})$

1. Ignore the fact that the uncertainty in the true value u is extreme.
2. Focus on the wild guess $\tilde{u}$.
3. Rank decisions according to their worst outcome on $U(\rho^*, \tilde{u})$.

This theory cannot escape the title "voodoo decision theory" so long as the neighborhood $U(\rho^*, \tilde{u})$ is relatively small in relation to $\mathscr{U}$.

To those readers who may counter that surely this would not be the case for a large $U(\rho^*, \tilde{u})$, I want to point out that an attempt to increase the size of this neighborhood may cause another type of complication. Since the theory is worst-case oriented, for an exceedingly large value of ρ^*, there might not be a $q \in Q$ such that $u \in A(q)$ for all u in $U(\rho^*, \tilde{u})$. In such cases all the decisions will be "unacceptable" in the worst-case sense, so that it will be hard to discriminate between robust and fragile decisions: all the decisions will be regarded as fragile over $U(\rho^*, \tilde{u})$.

[3]There are persistent claims that this theory is actually practiced in many organizations.

6.3 Radius of stability theory

Espousing as it is in the *Viva!* $1 \diamond 2 \diamond 3$ **Recipe**, this version has all the trimmings of a voodoo theory. Still in comparison to the above, this version has a better capability to discriminate between decisions when the value of ρ^* is set to the largest value of ρ such that $U(\rho, \tilde{u}) \subseteq A(q)$. The model it implements for the identification of robust decisions is the *radius of stability* model.

Definition 6.1 *Radius of stability.*
The largest value of ρ *such that* $U(\rho, \tilde{u}) \subseteq A(q)$ *is called the* $\boxed{\textit{radius of stability}}$ *of decision* q *at* $\tilde{u}$*. Symbolically:*

Radius of stability robustness model

$$\hat{\rho}(q, \tilde{u}) := \max_{\rho \geq 0} \{\rho : U(\rho, \tilde{u}) \subseteq A(q)\} \ , \ q \in Q \tag{16}$$

$$= \max_{\rho \geq 0} \{\rho : u \in A(q), \forall u \in U(\rho, \tilde{u})\}. \tag{17}$$

The larger the radius of stability $\hat{\rho}(q, \tilde{u})$, the more robust decision q locally at $\tilde{u}$.

In words: the radius of stability of decision q at $\tilde{u}$ is the radius ρ of the largest neighborhood $U(\rho, \tilde{u})$ around $\tilde{u}$ that is contained in $A(q)$. This is illustrated in Figure 9, where the radii of stability of two decision are shown.

It is important to take note, though, of the local orientation of this definition of robustness. In Figure 9 this is manifested in the fact that although $A(q')$ is much larger than $A(q'')$, the radius of stability of q' is much smaller than the radius of stability of q''. This is hardly surprising because, as indicated in the Preface, a decision that is locally robust around $\tilde{u}$ is not necessarily robust globally over $\mathscr{U}$, and vice versa.

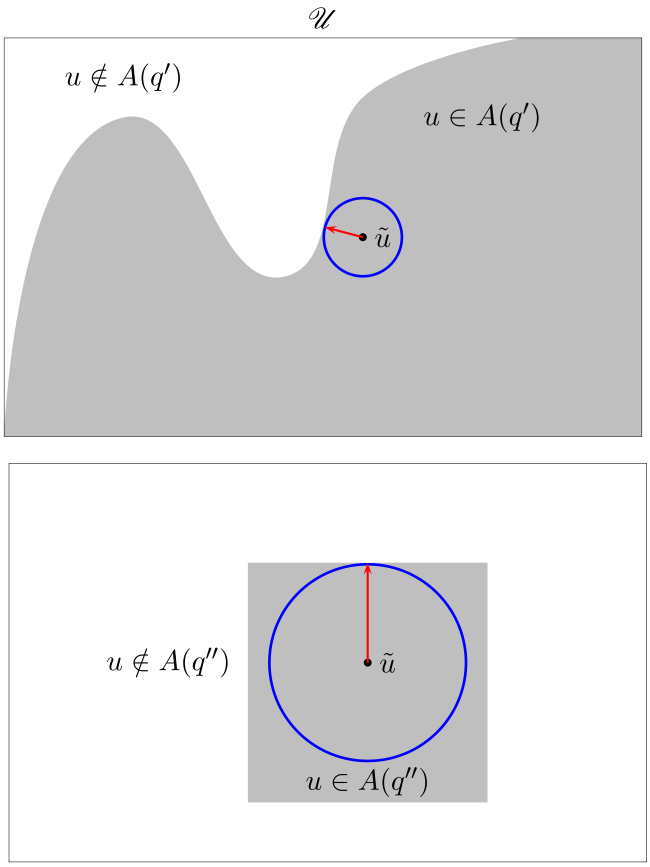

Figure 9: Radii of stability of two decisions at $\tilde{u}$

In short,

Radius of stability theory
1. Ignore the fact that the uncertainty in the true value u is extreme.
2. Focus on the wild guess $\tilde{u}$.
3. Rank decisions according to their radius of stability, $\hat{\rho}(q,\tilde{u})$, at $\tilde{u}$.

It is also important to note that the radius of stability based theory is in fact a local maximin type theory whose domain of analysis is not pre-determined. Hence, the radius of stability based version determines, for each decision q, the largest value of ρ, namely the largest neighborhood $U(\rho,\tilde{u})$ around $\tilde{u}$, such that the worst outcome over $U(\rho,\tilde{u})$ is equal to 1, namely such that $U(\rho,\tilde{u}) \subseteq A(q)$.

The radius of stability based theory can be regarded then as that instance of the local maximin based theory, were the pre-determined neighborhood corresponding to $\rho^* = \hat{\rho}(q,\tilde{u})$. Here, the best decision is one whose worst outcome $wo(q)$ is equal to 1 for the largest value of ρ^*. All this is grounded on the following facts:

Theorem 6.1 *The radius of stability robustness model (17) is a maximin model. Specifically, it is an instance of the following prototype maximin model, where $con(x,s)$ denotes a list of constraints on (x,s) pair:*

$$z^*(q,\tilde{u}) := \max_{x\in X} \min_{s\in S(x)} \{f(x,s) : con(x,s), \forall s \in S(x)\} \ , \ q \in Q \tag{18}$$

Proof. Consider the instance of (18) specified by

$$x \equiv \rho \ ; \ s \equiv u \ ; \ X = [0,\infty) \ ; \ S(x) \leftarrow U(\rho,\tilde{u}) \ ; f(x,s) \leftarrow \rho \ ; con(x,s) \leftarrow u \in A(q) \tag{19}$$

For this specification we have,

$$\max_{x\in X} \min_{s\in S(x)} \{f(x,s) : con(x,s), \forall s \in S(x)\} \equiv \max_{\rho\geq 0} \min_{u\in U(\rho,\tilde{u})} \{\rho : u \in A(q), \forall u \in U(\rho,\tilde{u})\} \tag{20}$$

$$\equiv \max_{\rho\geq 0} \{\rho : u \in A(q), \forall u \in U(\rho,\tilde{u})\}. \tag{21}$$

Hence, this instance is indeed the radius of stability robustness model (17). *QED*

6.4 Local Size-Robustness theory for a pre-determined neighborhood

This theory is a "localized version" the *Global Size-Robustness.* This is manifested in its robustness analysis being conducted on a neighborhood $U(\rho,\tilde{u})$ for a pre-determined value of ρ, rather than on $\mathscr{U}$. To see that this is so, set this pre-determined value of ρ to ρ^*, and regard it as the *radius* of the domain of the analysis.

Note that what is of concern here is the size of the subset of $U(\rho^*,\tilde{u})$ whose elements are acceptable values of u with respect to decision q. Thus, the robustness one deals with is *local.* Let then,

$$A(q,\rho^*,\tilde{u}) := U(\rho^*,\tilde{u}) \cap A(q) \tag{22}$$

$$= \{u \in U(\rho^*,\tilde{u}) : u \in A(q)\}. \tag{23}$$

The larger this set, the more robust decision q locally at $\tilde{u}$. Hence,

Definition 6.2 *Local Size-Robustness.*
The local size-robustness of decision $q \in Q$ at $\tilde{u}$ for a pre-determined radius ρ^* is as follows:

$$LSR(q,\rho^*,\tilde{u}) := size(A(q,\rho^*,\tilde{u})) \ , \ q \in Q \tag{24}$$

$$= \max_{V\subseteq U(\rho^*,\tilde{u})} \{size(V) : V \subseteq A(q)\} \tag{25}$$

$$= \max_{V\subseteq U(\rho^*,\tilde{u})} \{size(V) : u \in A(q), \forall u \in V\}. \tag{26}$$

The larger $LSR(q,\rho^*,\tilde{u})$, the more robust decision q locally at $\tilde{u}$.

This is illustrated in Figure 10. Note that the *Global Size-Robustness* of decision q' is much larger than the *Global Size-Robustness* of decision q'', whereas the *Local Size-Robustness* of decision q' is much smaller than the *Local Size-Robustness* of decision q''.

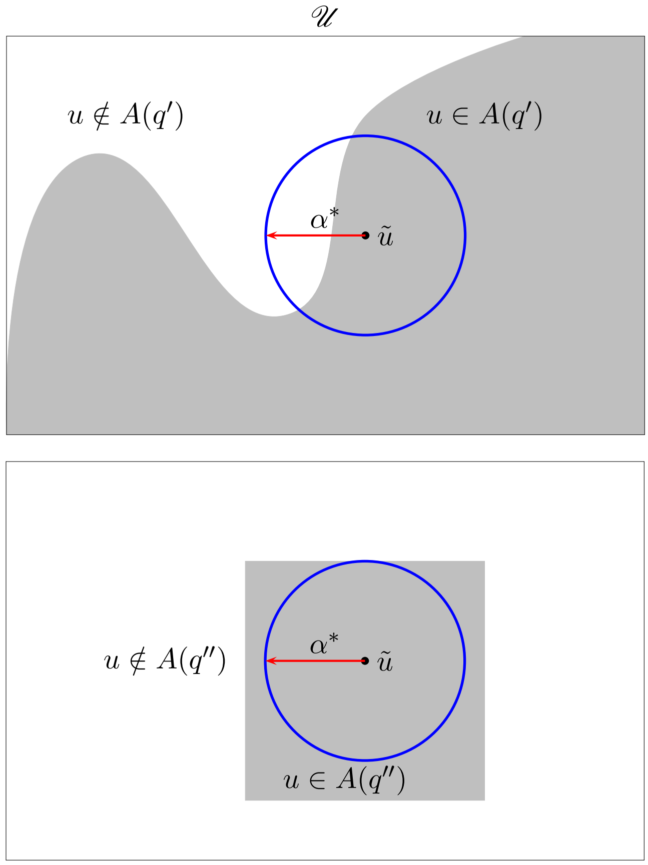

Figure 10: Illustration of the Local Size Robustness

This is hardly surprising because the fact that a decision is locally robustness around $\tilde{u}$ does not imply that it is globally robust over $\mathscr{U}$.

6.5 Viva! revisited

The *Viva!* $1 \diamond 2 \diamond 3$ **Recipe** apparently has its appeal. That this is so is evidenced by the fact that it had been adopted by a considerable number of risk analysts working in various disciplines. Its foremost attraction would seem to be that it reduces the formidable task of determining the robustness of decisions to *extreme* uncertainty into what appears to be a simple, uncomplicated recipe which can be developed into a number of voodoo decision theories.

The fact that this recipe does not address the challenging issues posed by *extreme* uncertainty, indeed that it effectively takes no notice of them at all, is of no concern when it comes to designing a voodoo theory. As I pointed out already, dodging the difficulties posed by *extreme* uncertainty is precisely what gives this recipe its voodoo character and this is the reason that I discuss it here in such detail.

7 And the winner is . . .

The four benchmark voodoo decision theories discussed in the preceding section are based on the same basic approach to *extreme* uncertainty, which puts forward the following recipe:

Viva! $1 \diamond 2 \diamond 3$ **Recipe**
Step 1. $\boxed{\text{Ignore}}$ the fact that the uncertainty in the true value u is extreme.
Step 2. $\boxed{\text{Focus}}$ on the wild guess $\tilde{u}$.
Step 3. Conduct a $\boxed{\text{local}}$ robustness analysis in the $\boxed{\text{neighborhood}}$ of $\tilde{u}$.

The difference between them derives from the model each implements, in Step 3, to perform the local robustness analysis around the wild guess $\tilde{u}$.

If we were to single out from among them the theory that most deserves the title *voodoo decision theory par excellence,* we would do this by consulting the following table:

Shades of voodooism	size of $\mathscr{D}(q)$ relative to the size of $\mathscr{U}$
plain	50% − 60%
crystal	40% − 50%
silver	30% − 40%
platinum	20% − 30%
gold	10% − 20%
par excellence	< 10%

recalling that $\mathscr{D}(q)$ denotes the domain of robustness analysis of decision q.

That is, we would seek to determine the "shade of voodooism" that each one of the four contenders has.

Observe then that this can be done conclusively only with regard to two theories, namely the *Mother of all voodoo decision theories* and the *radius of stability based theory.* The other two theories, namely the *local maximin theory for a pre-determined neighborhood* and the *Local Size-Robustness theory for a pre-determined neighborhood* are more difficult to pin down because they do not specify the size of the analysis domain $\mathscr{D}(q)$. This task is assigned to the USER of the theory, as it is the USER of the theory who determines the value of the radius ρ^* of the neighborhood $\mathscr{D}(q) = U(\rho^*, \tilde{u})$ on which the robustness analysis is conducted.

Hence, there are only two contenders for the title *voodoo decision theory par excellence.* Let us then examine them briefly.

7.1 Mother of all voodoo decision theories

This theory conducts its analysis on the neighborhood $U(0, \tilde{u})$, hence $\mathscr{D}(q) = \{\tilde{u}\}$, implying that $size(\mathscr{D}(q))=0$.

There is no doubt, therefore, that it would easily qualify for the title *voodoo decision theory par excellence.* The trouble is, though, that I cannot provide references to peer-reviewed articles in which this theory is advocated for the treatment of *extreme* uncertainty[4].

7.2 Radius of stability based theory

This theory does not specify the size of $\mathscr{D}(q)$ in advance. This size is determined in the course of the analysis as prescribed by the radius of stability robustness model, namely by:

$$\hat{\rho}(q, \tilde{u}) := \max_{\rho \geq 0} \{\rho : u \in A(q), \forall u \in U(\rho, \tilde{u})\} \ , \ q \in Q. \tag{27}$$

More precisely, $\mathscr{D}(q) = U(\rho^*, \tilde{u})$, where $\rho^* = \hat{\rho}(q, \tilde{u}) + \varepsilon$, and ε can be arbitrarily small (but positive).

Since the theory puts no restrictions on which value of ρ^* the robustness model can use, small values of ρ^* are not only perfectly legitimate, they are treated as thoroughly informative. Thus, according to this theory, a minutely small value of $\hat{\rho}(q, \tilde{u})$ is taken as evidence that decision q is very $\boxed{\text{fragile.}}$

[4] I would greatly appreciate any hints regarding the existence of such peer-reviewed articles.

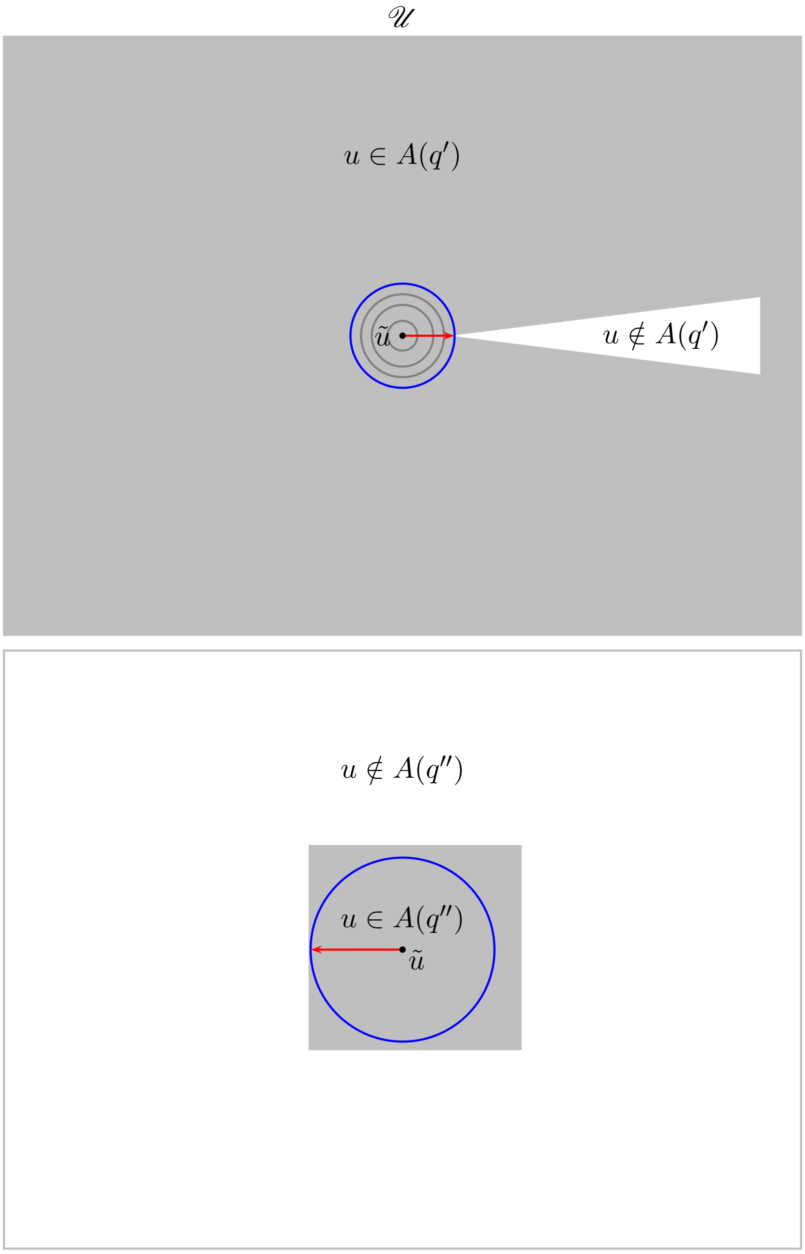

Figure 11: Radii of stability of two decisions at $\tilde{u}$

This is illustrated in Figure 11, where the radii of stability of two decisions are shown. According to the *radius of stability* based theory, decision q' is very fragile, compared to decision q''.

In short, not only that the theory does not a priori restrict the size of its domain of analysis, it draws critically important conclusions in cases where this domain is extremely small.

The implication is therefore that this theory would have no qualms to accept as legitimate, domains of analysis whose size is less than 10% of the size of the uncertainty space $\mathscr{U}$. And the conclusion to be drawn from this is that this theory amply deserves the title *voodoo decision theory par excellence*, especially in cases where the uncertainty space is unbounded.

For the record therefore:

Theorem 7.1 *A decision theory employing the radius of stability model for the purpose of decision under extreme uncertainty, is a voodoo decision theory par excellence, especially in cases where the uncertainty space is unbounded.*

Proof. The radius of stability based theory has no qualms whatsoever to draw critically important conclusions about the robustness of decisions and about robustness-based comparisons of decisions, even in cases where the radius of stability is extremely small (say, less than 10% of the size of the uncertainty space). In cases where the uncertainty space is unbounded and the robustness of decisions is bounded, the size of the domain of the robustness analysis can be infinitesimally small, compared to the size of the uncertainty space. *QED*

The pathologic case concerns a decision q^* such that $A(q^*) = \mathscr{U} \setminus \{\tilde{u}\}$, that is a decision that satisfies the performance constraint everywhere on $\mathscr{U}$ except at $\tilde{u}$. The *radius of stability based theory* judges this decision to be extremely fragile, in fact as fragile as a decision that violates the performance constraint over the entire uncertainty space!!!

It is patently clear that this theory is fundamentally flawed and that it most certainty deserves to be a contender for the title of this competition.

Conclusion

On the face of it, the clear winner here should be the *Mother of all voodoo theories.* However, in the absence of references to peer-reviewed articles advocating the use of the *Mother of all voodoo theories* as a tool for the treatment of *extreme* uncertainty, the radius of stability based theory emerges as the clear winner. This theory is most definitely a *voodoo decision theory par excellence,* especially when applied in cases where the uncertainty space is unbounded. It is inexplicable therefore that, it is being advocated as a tool for the management of extreme uncertainty in peer reviewed journals, such as *Risk Analysis.*

8 Voodoo decision-making info-gap style

According to info-gap decision theory (Ben-Haim 2001, 2006, 2010), the robustness of decision q is defined as follows:

Info-gap robustness model:

$$\hat{\rho}(q,\tilde{u}) := \max_{\rho\geq 0} \{\rho : r_c \leq r(q,u), \forall u \in U(\rho,\tilde{u})\} \ , \ q \in Q \tag{28}$$

where $r(q,u)$ denotes the *performance level* of decision q given u and r_c denotes the *critical performance level* under consideration.

In view of the above, the info-gap robustness of decision q at $\tilde{u}$ is the radius of stability of decision q at $\tilde{u}$, where the set of acceptable values of u associated with decision q is as follows:

$$A(q) := \{u \in \mathscr{U} : r_c \leq r(q,u)\} \ , \ q \in Q. \tag{29}$$

8.1 Robust-satisficing a la info-gap

According to info-gap's *robust-satisficing approach,* the larger the robustness the better. Hence, the best (optimal) decision is one that solves this optimization problem:

Info-gap robust-satisficing decision model:

$$\hat{\rho}(\tilde{u}) := \max_{q\in Q} \hat{\rho}(q,\tilde{u}) \tag{30}$$

$$= \max_{q\in Q}\max_{\rho\geq 0} \{\rho : r_c \leq r(q,u), \forall u \in U(\rho,\tilde{u})\} \tag{31}$$

$$= \max_{q\in Q,\rho\geq 0} \{\rho : r_c \leq r(q,u), \forall u \in U(\rho,\tilde{u})\}. \tag{32}$$

One may well ask therefore: how is it then that there is no mention whatsoever of the *radius of stability* connection in info-gap's primary texts (Ben-Haim 2001, 2006, 2010)?

Indeed, one may ask the same question about Wald's maximin model given that info-gap's robustness model and decision model are simple instances of Wald's maximin model. But, not only that these facts are not even noted in info-gap's primary texts, info-gap decision theory is proclaimed in them as new and radically different from all current theories of decision under uncertainty.

It should be pointed out, therefore, that all that is new, and radically different in info-gap decision theory is the proposition that a radius of stability model such as (28) is a suitable tool for the management of *extreme* uncertainty.

This proposition is indeed radically different in that it is unique. There seems to be no other decision theory that has made this proposition. In any event, no other theory has made this proposition from the pages of peer-reviewed journals, such as *Risk Analysis*.

For the record then,

Theorem 8.1 *Info-gap's robustness model is a simple radius of stability model, hence a simple maximin model.*

Proof. By inspection, Info-gap's robustness model is the simple radius of stability model where set $A(q)$ is defined/specified as follows: $A(q) = \{u \in \mathscr{U} : r_c \leq r(q,u)\}$. It therefore follows from Theorem 6.1 that info-gap's robustness model is maximin model, specifically it is an instance of the prototype maximin model given in (18). *QED*

Theorem 8.2 *Info-gap's robust-satisficing decision model is a simple maximin model. Specifically, it is a simple instance of the prototype maximin model given in (18).*

Proof. The proof runs along the same lines as that of Theorem 6.1, except that the instance under consideration is as follows:

$$x \equiv (q,\rho)\ ;\ s \equiv u\ ;\ X = Q \times [0,\infty)\ ;\ S(x) \leftarrow U(\rho,\tilde{u})\ ; f(x,s) \leftarrow \rho\ ;\ con(x,s) \leftarrow r_c \leq r(q,u) \quad (33)$$

Substituting this in (18), we obtain

$$\max_{x\in X}\min_{s\in S(x)} \{f(x,s) : con(x,s), \forall s \in S(x)\} \equiv \max_{q\in Q,\rho\geq 0}\min_{u\in U(\rho,\tilde{u})} \{\rho : r_c \leq r(q,u), \forall u \in U(\rho,\tilde{u})\} \quad (34)$$

$$\equiv \max_{q\in Q,\rho\geq 0} \{\rho : r_c \leq r(q,u), \forall u \in U(\rho,\tilde{u})\}. \quad (35)$$

Hence, this instance is indeed info-gap's robust-satisficing decision model(32). *QED*

Theorem 8.3 *As a method for decision-making under extreme uncertainty, info-gap decision theory is a voodoo decision theory par excellence, especially in cases where the uncertainty space is unbounded.*

Proof. This is an immediate implication of Theorem 7.1, observing that info-gap's robustness model is a radius of stability model. *QED*

In view of what we saw thus far, the question naturally arising is this:

- How is it then that this theory has managed to pass muster in the revered peer-review process of professional journals, such as *Risk Analysis*?

As might be expected I do not have a definitive answer to this intriguing question. What I can confidently state, though, is that the factor that had no doubt made this possible is the rhetoric in the info-gap literature about the severity of the uncertainty that the theory claims to address, the type of robustness that the theory purportedly obtains, the theory's scope of application, and so on.

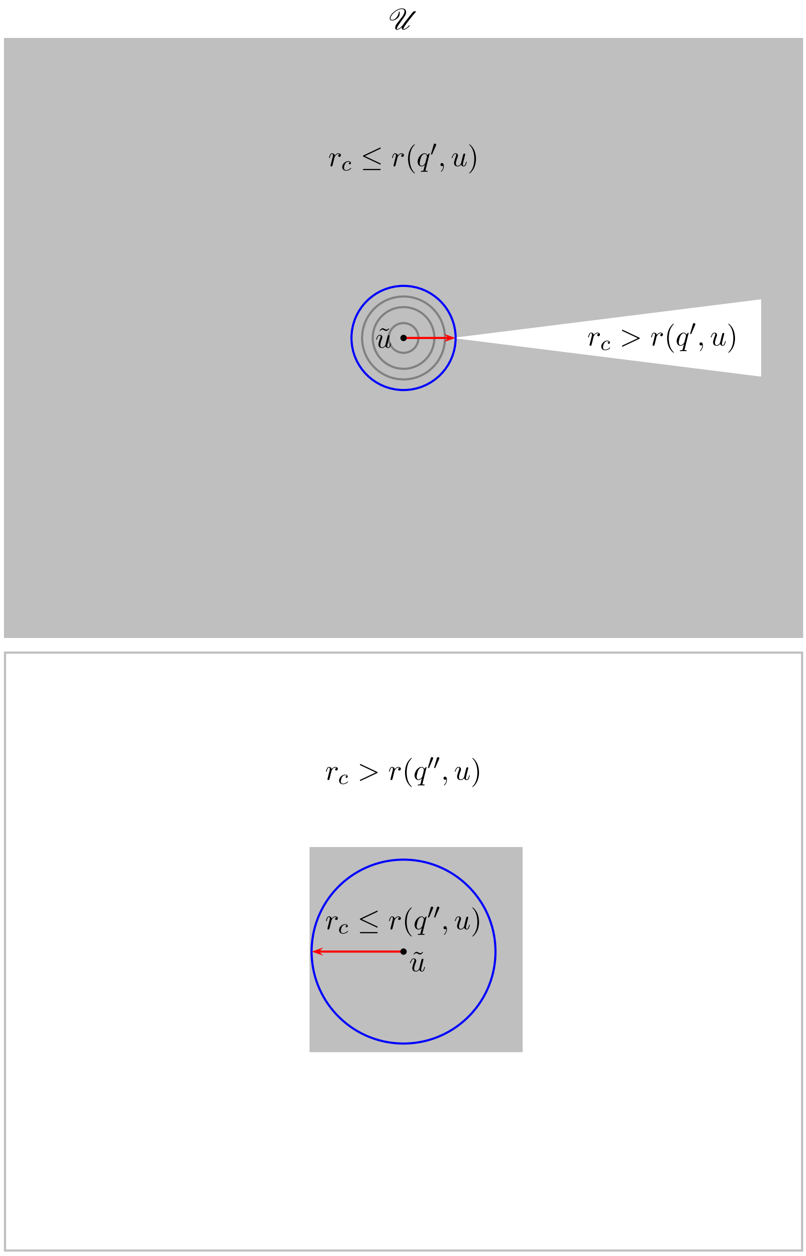

Figure 12: Info-gap robustness of two decisions at $\tilde{u}$

8.2 Is info-gap robustness local?

The inherently local nature of info-gap's robustness model is illustrated in Figure 12. It shows the info-gap robustness of two decisions, q' and q'', where the shaded areas represent the sets of acceptable values of u associated with these decisions. Clearly, the info-gap robustness of decision q'' is much larger than the info-gap robustness of decision q', even though the set of acceptable values of q' is much larger than the set of acceptable values of q''.

To have a correct picture of what info-gap robustness is, simply view the values of $u \in \mathscr{U}$ as perturbations/deviations in the value of $\tilde{u}$. Robustness is thus treated as a measure of the perturbations/deviations in the value of $\tilde{u}$. The local nature of info-gap robustness is manifested in the fact that info-gap's robustness model is concerned **first and foremost** with small perturbations/deviations in the value of $\tilde{u}$. Thus, the info-gap robustness of decision q is a robustness against small perturbations/deviations in the value of $\tilde{u}$. This means that a large perturbation/deviation in the value of $\tilde{u}$ is "acceptable" only if all smaller perturbations/deviations are "acceptable". This is illustrated in Figure 13 which features two decisions q' and q''. Note that although decision q' is highly robust against large perturbations/deviations in the value of $\tilde{u}$, its info-gap robustness is small because this decision is not robust against small perturbations/deviations in $\tilde{u}$.

In contrast, decision q'' is highly fragile against large perturbations/deviations in $\tilde{u}$, but it is much more robust than decision q' against small perturbations/deviations in the value of $\tilde{u}$. But according to the precepts of info-gap decision theory, decision q'' is deemed much more robust than decision q'.

And this determination is to be expected because, like all radius of stability models, info-gap's robustness model seeks the smallest perturbation/deviation in the value of $\tilde{u}$ that will violate the performance constraint if increased slightly, observing that

$$\hat{\rho}(q,\tilde{u}) := \max_{\rho\geq 0} \{\rho : r_c \leq r(q,u), \forall u \in U(\rho,\tilde{u})\} \tag{36}$$

$$= \min_{\rho\geq 0} \{\rho : r_c > r(q,u), \forall u \in U(\rho',\tilde{u}), \rho' > \rho\} \tag{37}$$

So, of crucial importance in the info-gap robustness analysis of decision q is the *location* of the values of u that violate the performance constraint, relative to the location of $\tilde{u}$. Roughly, the closer this value is to $\tilde{u}$, the more fragile q.

In short, methodologically, it is instructive to think of info-gap's robustness model as a model that is concerned, in the first place, with small perturbations. If a small perturbation violates the performance constraint, then no larger perturbations are considered. To wit: if a perturbation of size $\rho = 0$, namely if $u = \tilde{u}$, violates the performance constraint $r_c \leq r(q,u)$, then the info-gap robustness of decision q is set to $\hat{\rho}(q,r_c) = 0$, regardless of the performance of the decision over $\mathscr{U} \setminus \{\tilde{u}\}$. This is a *local* property par excellence.

Last but not least, the most obvious manifestation of the local orientation of info-gap's robustness model is, of course, the fact that the info-gap robustness of decision q may depend on the value of $\tilde{u}$, and may thus vary significantly as the value of $\tilde{u}$ varies. This is illustrated in Figure 14, where the info-gap's robustness of decision q is shown for six values of $\tilde{u}$. Note that the info-gap robustness of decision q is equal to zero for $\tilde{u}_1$ and $\tilde{u}_2$.

It is important to take note of this fact because info-gap decision theory regards $\tilde{u}$ as a poor guess of the true value of u, even a wild guess. Yet, the theory allows this poor/wild guess to play a central role in the robustness analysis, to thereby have a critical impact on the value of $\hat{\rho}(q,r_c)$.

9 From robust reliability in the mechanical sciences to decision-making under severe uncertainty

The foregoing discussion makes it abundantly clear that info-gap's robustness model is a model of local robustness. Still, it should prove edifying to demonstrate this fact from a slightly different

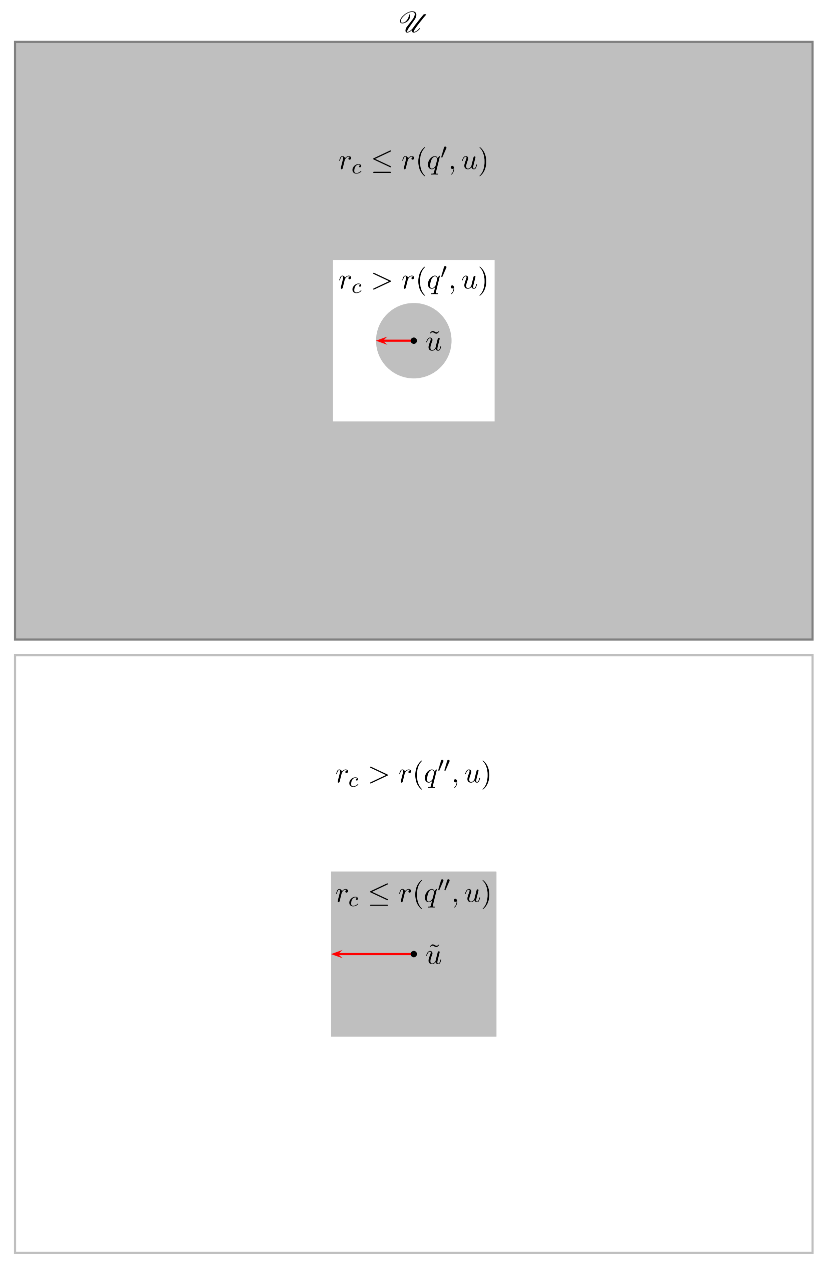

Figure 13: Info-gap's robustness: small vs large perturbations/deviations

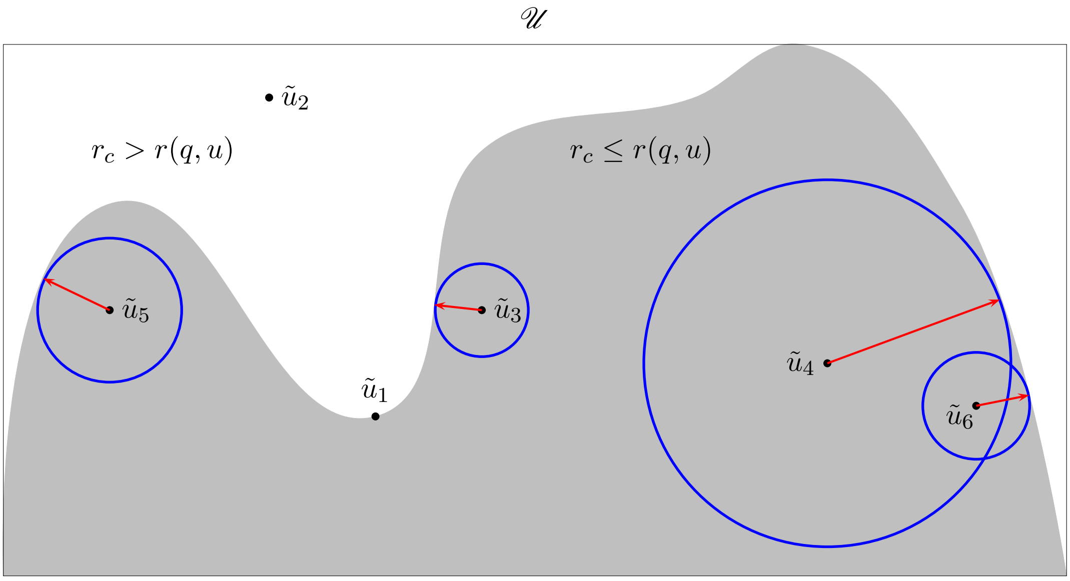

Figure 14: Info-gap robustness of q for six values of $\tilde{u}$

angle, which has to do with the evolution of this theory from a *robust reliability theory* into a *decision theory* for coping with severe uncertainty. Observe then that the origins of info-gap decision theory can be traced back to the book *Robust reliability in the mechanical sciences* (Ben-Haim 1996), where a quick scan of the robustness models featured in this book immediately reveals that these models are of a piece with info-gap's robustness model. Consider what the basic concern of these robustness models is:

> In using a convex set to represent uncertainty we refer to the size parameter, such as α in eq. (2.21), as a *measure of uncertainty.* In the geometry of convex sets the size parameter is called the *expansion parameter* of the set. The set $\mathcal{U}(\alpha)$ expands and contracts like a ballon as α grows and diminishes, retaining its shape but varying its dimensions. In robust reliability analysis we will often ask: how much can the convex model of uncertainty expand before failure becomes possible.
>
> Ben-Haim (1996, p. 16)

Note: eq. (2.21) is as follows:

$$\mathcal{U}(\alpha) = \left\{u(t) : [u(t) - \overline{u}(t)]^T[u(t) - \overline{u}(t)] \leq \alpha^2\right\} \qquad (2.21)$$

Clearly, this understanding of robustness is captured in the **local** **radius of stability** concept. Because, the question: how much can the balloon expand before it fails (blows up?), depends very much on the balloon's location in the geometric space under consideration. And, to answer it, one seeks to identify the **smallest deviation** from the balloon's center-point (= nominal value of the parameter) that can cause failure.

My point is then that other than neglecting to make it clear to the readers that the robustness models developed in Ben-Haim (1996) are in fact typical radius of stability models (that go back to the 1960's), the analysis in Ben-Haim (1996) is on the whole sound. That is, the radius of stability type models that are discussed in this book are deployed to perform the task that they had been envisioned for, which is: determining the robustness of systems against small perturbations in the nominal value of a parameter.

Furthermore, there is nothing in the analysis in Ben-Haim (1996) to suggest that these robustness models are designed for, or are suitable for, situations that are subject to a **severe** uncertainty of the type stipulated in Ben-Haim (2001, 2006, 2010). Indeed, consider this:

> Finally, a reliability theory is only as good as the information upon which it rests. A reliability theory should exploit all relevant verified information, but should treat speculative information and "reasonable assumptions" with caution.
>
> Ben-Haim (1996, p. 206)

Presumably, this warning applies, among other things, to the employment of a wild guess as the center-point of a local analysis, particularly to the claims that such an analysis is reliable.

There are definitely no claims in Ben-Haim (1996) that these models are particularly suitable for situations where the uncertainty parameter is unbounded.

And there are definitely no claims in Ben-Haim (1996) that these models are radically different from all other non-probabilistic robustness models known at that time.

One wonders, therefore, on what grounds did a theory that was based on the concepts and models of local robustness that figured in the discussion on reliability in Ben-Haim (1996) metamorphose, five years later, into a **new decision theory** that is **radically different from all** current theories for decision under **severe** uncertainty.

I call attention to the fact that in its 1996 incarnation, info-gap decision theory, most definitely was neither envisioned, nor presented, as a theory for the treatment of a severe uncertainty of the type stipulated in Ben-Haim (2001, 2006, 2010), to demonstrate that the main difference between the analyses in Ben-Haim (1996) and Ben-Haim (2001, 2006, 2010) is in the rhetoric.

The rhetoric in Ben-Haim (2001, 2006, 2010) lays great stress on the challenges posed by severe uncertainty; and the robustness models in these texts are claimed to be particularly suitable for the treatment of a severe uncertainty that is characterized by an unbounded uncertainty space, a poor point estimate and a likelihood-free quantification of uncertainty. But these claims in Ben-Haim (2001, 2006, 2010) are not corroborated by proofs, and/or demonstrations, and/or experiments.

It is important, therefore, to acquaint the reader with the rhetoric and spin that saturate the info-gap literature, particularly because they continue to elude referees and editors of peer-reviewed journals.

10 Rhetoric info-gap style

In this section I survey and discuss the rhetoric that pervades the info-gap literature focusing on those topics that are central to a correct assessment of this theory.

10.1 Back to basics

One of most important examples of info-gap rhetoric is the rhetoric in the info-gap literature surrounding the *definition of info-gap robustness.* This definition is clear cut so that there are no two ways about it: the robustness sought by info-gap decision theory is *local.* That is, info-gap robustness is a measure of *local* robustness.

Recall that the generic form of this definition is as follows (I use α instead of ρ to comply with the standard notation in the info-gap literature (Ben-Haim 2001, 2006)):

$$\hat{\alpha}(q, r_c) := \max_{\alpha \geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha, \tilde{u})\} \ , \ q \in Q. \tag{38}$$

In words: the robustness of decision q, denoted $\hat{\alpha}(q, r_c)$, is equal to the size (α) of the largest neighborhood ($U(\alpha, \tilde{u})$) around $\tilde{u}$ all of whose elements (u) satisfy the performance constraint $r_c \leq r(q, u)$.

The transparent mathematical structure of info-gap's robustness model leaves no room for debates about the nature of this model. Like all radius of stability models, this is a model of *local* robustness par excellence.

And yet the info-gap literature is saturated with denials that it is a model of *local robustness* for instance: "…Info-gap theory uses the analyst's models, but this does not make it a "local" theory of robustness." (Ben-Haim 2012a, p. 7), and with spin that misrepresents the facts, for instance: "…The analysis of a continuum of uncertainty from local to global is one of the novel ways in which info-gap analysis is informative." (Hall et al. 2012, p. 6).

All this spin and rhetoric is required to support the absurd proposition that this model offers a reliable model of robustness against *extreme* uncertainty, namely a model capable of dealing with large perturbations/deviations in $\tilde{u}$.

Similar denials and obfuscations abound about the fact that info-gap's robustness model and info-gap's robust-satisficing decision model are maximin models.

It is important to keep in mind therefore that contrary to the info-gap-lore, info-gap's robustness model is neither new, nor radically different from main stream, well established, indeed famous models of robustness. In fact, as I have shown above, it is a reinvention of the famous *radius of stability model* (circa 1960):

$$\hat{\rho}(q,\tilde{u}) := \max_{\rho \geq 0} \{\alpha : u \in A(q), \forall u \in U(\alpha,\tilde{u})\} \ , \ q \in Q. \tag{39}$$

Furthermore, both models are very simple instances of Wald's famous maximin model (circa 1940):

$$z^* := \max_{x \in X} \min_{s \in S(x)} \{f(x,s) : con(x,s), \forall s \in S(x)\} \tag{40}$$

where $con(x,s)$ denotes a list of constraints on the (x,y) pairs.

The trouble of course is that all this continues to escape the referees of peer-reviewed journals such as *Risk Analysis.*models.

I discussed the rhetoric in the info-gap literature about the maximin model in the article *Rhetoric in risk analysis, Part I: Wald's mighty maximin paradigm* (Sniedovich 2012d), so I need not repeat the discussion on this subject here.

What I want to demonstrate here, is the centrality of the rhetoric in the info-gap literature. To do this, I need do no more than quote extracts from this literature and briefly comment on them. As you shall see, these speak for themselves ...

10.2 The big picture

First consider the statements that introduced info-gap decision theory to the world in 2001, as well as more recent ones.

> Info-gap decision theory is radically different from all current theories of decision under uncertainty. The difference originates in the modeling of uncertainty as an information gap rather than as a probability. The need for info-gap modeling and management of uncertainty arises in dealing with severe lack of information and highly unstructured uncertainty.
>
> Ben-Haim (2001, 2006, p. xii)

> In this book we concentrate on the fairly new concept of information-gap uncertainty, whose differences from more classical approaches to uncertainty are real and deep. Despite the power of classical decision theories, in many areas such as engineering, economics, management, medicine and public policy, a need has arisen for a different format for decisions based on severely uncertain evidence.
>
> Ben-Haim (2001, 2006, p. 11)

> Probability and info-gap modelling each emerged as a struggle between rival intellectual schools. Some philosophers of science tended to evaluate the info-gap approach in terms of how it would serve physical science in place of probability. This is like asking how probability would have served scholastic demonstrative reasoning in the place of Aristotelian logic; the answer: not at all. But then, probability arose from challenges different from those faced the scholastics, just as the info-gap decision theory which we will develop in this book aims to meet new challenges.
>
> Ben-Haim (2001 and 2006, p. 12)

> The emergence of info-gap decision theory as a viable alternative to probabilistic methods helps to reconcile Knights dichotomy between risk and uncertainty. But more than

> that, while info-gap models of severe lack of information serve to quantify Knights unmeasurable uncertainty, they also provide new insight into risk, gambling, and the entire pantheon of classical probabilistic explanations. We realize the full potential of the new theory when we see that it provides new ways of thinking about old problems.
>
> Ben-Haim (2001 p. 304; 2006, p. 342)

> Info-gap decision theory clearly presents a 'replacement theory' with which we can more fully understand the relation between classical theories of uncertainty and uncertain phenomena themselves.
>
> Ben-Haim (2001 p. 305; 2006, p. 343)

> The management of surprises is central to the "economic problem", and info-gap theory is a response to this challenge. This book is about how to formulate and evaluate economic decisions under severe uncertainty. The book demonstrates, through numerous examples, the info-gap methodology for reliably managing uncertainty in economics policy analysis and decision making.
>
> Ben-Haim (2010, p. x)

These are extremely bold statement. So, the question naturally is:

- On what grounds are these claims made?

The fact of the matter is that there is nothing in the info-gap literature to substantiate these claims, let alone "prove", that they are indeed valid. If anything, these can be easily shown to be groundless and misleading. Because, all one needs to do to this end is to show that info-gap's robustness model and info-gap's robust-satisficing are simple instances of Wald's famous maximin model. Furthermore, that info-gap robustness is a re-invention of the well familiar concept *radius of stability* (circa 1960).

If anything, these claims demonstrate a profound lack of familiarity with the state of the art.

10.3 About the maximin connection

The same goes for assertions about the maximin connection:

> The difference from min-max approaches is that we are able to select a policy without ever specifying how wrong the model actually is. Min-max and info-gap robust-satisficing strategies will sometimes agree and sometimes differ.
>
> Ben-Haim (2010, p. 10)

> While there is a superficial similarity with minimax decision making, no fixed bounds are imposed on the set of possibilities, leading to a comprehensive search of the set of possibilities and construction of functions that describe the results of that search.
>
> Hine and Hall (2010, p. 17)

> Info-gap generalizes the maximin strategy by identifying worst-case outcomes at increasing levels (horizons) of uncertainty. This permits the construction of 'robustness curves' that describe the decay in guaranteed minimum performance (or worst-case outcome) as uncertainty increases.
>
> Wintle et al. (2011, p. 357)

> These two concepts of robustness—min-max and info-gap—are different, motivated by different information available to the analyst. The min-max concept responds to severe uncertainty that nonetheless can be bounded. The info-gap concept responds to severe uncertainty that is unbounded or whose bound is unknown. It is not surprising that min-max and info-gap robustness analyses sometimes agree on their policy recommendations, and sometimes disagree, as has been discussed elsewhere.(40)

Ben-Haim (2012, p. 7)
(40) = Ben-Haim et al. (2009).

> The relation between min-max and info-gap robust-satisficing has been discussed at length elsewhere.[12] The two methods have much apparent similarity, though also important differences. Most significantly, they depend on different prior information, and can lead to different solutions. Briefly, min-max requires knowledge of a worst case. In contrast, the horizon of uncertainty of an info-gap model is unknown and unbounded, thus deliberately avoiding the specification of a worst case. On the other hand, the info-gap robustness does require the analyst to specify the worst acceptable outcome, which in engineering design is usually a design specification.

Ben-Haim (2012a, p. 9)
[12] = Ben-Haim et al. (2009).

As I pointed out already, the formal, rigorous proofs that info-gap's robustness model and info-gap's robust-satisficing decision model are maximin models, which I should add have been in the pubic domain since 2007, show this rhetoric for what it is.

10.4 Non-probabilistic, likelihood-free quantification of uncertainty

A big fuss is made in the info-gap literature that info-gap decision theory is non-probabilistic, likelihood-free, belief-free, and so on. This means that the theory is claimed to deal with situations where there are no grounds to assume that the true value of u is more/less likely to be in the neighborhood of any particular value of u. Specifically, where no reasons exist to assume that the true value of u is more/less likely to be in the neighborhood of $\tilde{u}$.

> However, unlike in a probabilistic analysis, r has no connotation of likelihood. We have no rigorous basis for evaluating how likely failure may be; we simply lack the information, and to make a judgment would be deceptive and could be dangerous. There may definitely be a likelihood of failure associated with any given radial tolerance. However, the available information does not allow one to assess this likelihood with any reasonable accuracy.

Ben-Haim (1994, p. 152)

> Uncertainty is the potential for deviation of an actual realization from its normative form. Neither norm nor any specific potential realization is uncertain; it is the potential for deviation of one from the other which is info-gap uncertainty.
>
> The spatial analogy for info-gap uncertainty demonstrates that we need no concept of chance, frequency of recurrence, likelihood, plausibility or belief in order to speak of uncertainty.

Ben Haim (2001, p. 18; 2006, p. 22)

> Info-gap models are axiomatically utterly different from both probability and fuzzy logic, since info-gap models focus on the set-structure of uncertainty rather than on measure-theoretical representations. Info-gap models are particularly suited to representing sparse information since they make no assertions about frequencies of, or beliefs about, rare events.

Ben-Haim Y (2002, November 5)
Quote from the abstract of a seminar at MIT entitled: Info-Gap Decision Theory For Design And Planning Or: Why 'Good' Is Preferable To 'Best'.

> Info-gap models are axiomatically utterly different from both probability and fuzzy logic, since info-gap models focus on the set-structure of uncertainty rather than on measure-theoretical representations. Info-gap models are particularly suited to representing sparse information since they make no assertions about frequencies of, or beliefs

about, rare events.

Ben-Haim Y (2003, June 5)

Quote from the abstract of a seminar at Los Alamos National Lab entitled: Info-gap decision theory for design and planning.

Since the horizon of uncertainty is unknown and unbounded, there is no worst case. Since no measure functions of probability (or plausibility, or belief, etc.) are specified by an info-gap model, the analyst cannot calculate statistical expectations and cannot probabilistically insure against the unknown contingencies identified in the info-gap model.

Ben-Haim Y. and Jeske, K. (2003, p. 12)

And consider this:

However, unlike in a probabilistic analysis, r has no connotation of likelihood. We have no rigorous basis for evaluating how likely failure may be; we simply lack the information, and to make a judgment would be deceptive and could be dangerous. There may definitely be a likelihood of failure associated with any given radial tolerance. However, the available information does not allow one to assess this likelihood with any reasonable accuracy.

Ben-Haim (1994, p. 152)

The question is then, if this is indeed so, what are the grounds to designate a poor guess $\tilde{u}$ as the fulcrum of the robustness analysis; moreover, to proclaim the local robustness analysis in the neighborhood of this poor guess $\tilde{u}$ a reliable measure of the robustness of decisions against variations in the value of u over the vast (unbounded) uncertainty space $\mathscr{U}$? In other words, what is the logic behind the proposition to deal with such a severe uncertainty by means of a local robustness analysis around a poor/wild guess?

The short answer is: None!

Having finally noticed this glaring incongruity in the theory, some info-gap adherents tried to correct it. The trouble is however, that the cure turned out to be no better than the disease:

An assumption remains that values of u become increasingly unlikely as they diverge from $\tilde{u}$.

Hall and Harvey (2009, p. 2)

The point is, of course, that this "assumption" completely contradicts the very spirit of info-gap decision theory, namely its being a non-probabilistic, likelihood-free theory. Furthermore, suppose that $\mathscr{U}$ is the real line and that the uncertainty is quantified by a normally distributed random variable whose mean is $\tilde{u} = 0$ and whose variance is $\sigma^2 = 100$, as shown in Figure 15. Clearly, this case satisfies Hall and Harvey's (2009) assumption.

However, this can hardly justify a local robustness analysis in a small neighborhood around $\tilde{u} = 0$ that completely ignores the performance of decisions outside this neighborhood.

In a nutshell, info-gap adherents are perfectly happy to use a non-probabilistic, likelihood-free theory that conducts its local robustness analysis in the neighborhood of a poor guess, as a reliable tool for the treatment of severe uncertainty whose uncertainty space is unbounded! Furthermore, they have no qualms to ascribe this theory the extraordinary power to create probabilities, beliefs, likelihood, reliability and so on, out of thin air, or more accurately by a stroke of a pen:

One possible approach would be the application of tools from non-probabilistic decision theories, such as info-gap decision theory (Ben-Haim, 2006). Whereas classical decision theory approaches generally optimise the expected value of the decision variable, the info-gap approach instead minimises the probability of falling below a certain threshold (i.e., it maximises robustness to failure).

Chisholm (2010, p. 1981)

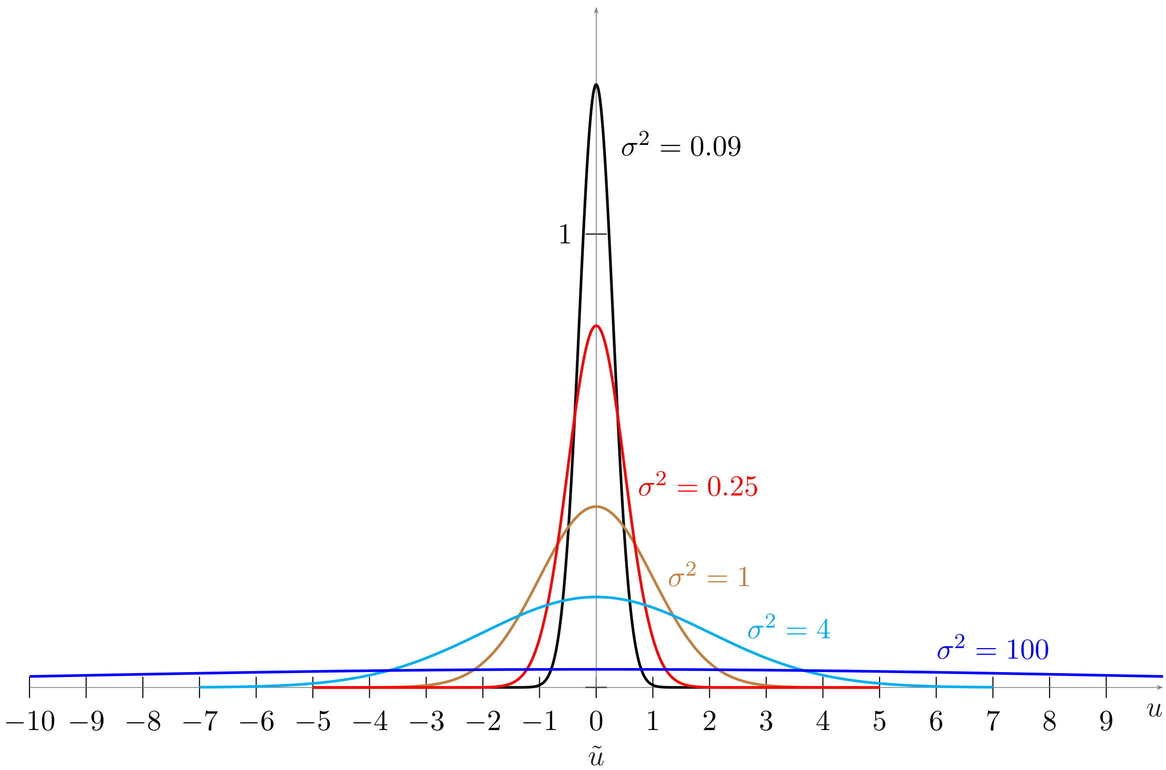

Figure 15: Normal distribution $N(\tilde{u}, \sigma^2)$, with mean $= \tilde{u} = 0$

How can a non-probabilistic decision theory, such as info-gap decision theory, possibly minimize the probab of falling below a certain threshold?

And how about this claim regarding the capabilities of info-gap decision theory?

> For example, the best management option may be one that ensures that a species does not exceed a given risk of extinction under the highest possible level of unfavorable uncertainty. The decision may not minimize the extinction risk when uncertainty is ignored, but it is the option least likely to fail because of uncertainty in model structure or parameter estimates.
>
> Nicholson and Possingham (2007, p. 252)

How can a likelihood-free model, such as info-gap's robustness model, possibly identify an option that is least likely to fail?

And along the same lines, consider the following quote from *ACERA Endorsed Core Material*[5]:

> Information-gap (henceforth termed 'info-gap') theory was invented to assist decision-making when there are substantial knowledge gaps and when probabilistic models of uncertainty are unreliable (Ben-Haim 2006). In general terms, info-gap theory seeks decisions that are most likely to achieve a minimally acceptable (satisfactory) outcome in the face of uncertainty, termed robust satisficing. It provides a platform for comprehensive sensitivity analysis relevant to a decision.
>
> Burgman, Wintle, Thompson, Moilanen, Runge, and Ben-Haim (2008, p. 8)

How can a likelihood-free theory possibly seek decisions that are most likely to achieve a minimally acceptable (satisfactory) outcome in the face of uncertainty?

The answer to these intriguing, questions is simple. In the realm of a *Voodoo theory* such obvious, fundamental contradictions are not an issue.

Next, let us take a closer look at the rhetoric denying that info-gap decision theory is based on a model of *local* robustness.

[5] See http://www.acera.unimelb.edu.au/materials/core.html

10.5 Local robustness

Interestingly, the whole issue of *local* robustness and info-gap decision theory is not even touched on in the three books on the theory (Ben-Haim 2001, 2006, 2010). In response to my criticism, this issue was taken up in Ben-Haim's (2007) working paper dealing with FAQs about info-gap decision theory, where the first FAQ is as follows:

> **1. Does an Info-Gap Model only Deal with Local Uncertainty?**
>
> **Question:**
>
> The best estimate, $\tilde{u}$, of an info-gap model of uncertainty is sometimes a wild guess, since in most cases the horizon of uncertainty, α, is unknown. How sure can we be that an info-gap model of uncertainty $U(\alpha, \tilde{u})$ is not just a local analysis of risks which grossly errs in the true value u? Is it not preferable to employ qualitative methods for managing"unknown-unknowns"? Does the info-gap approach simply sweep major risks under the carpet?
>
> Ben-Haim (2007, p. 2)

Ben-Haim's (2007) answer to this question consists of seven parts. The only part that actually deals with the *local* issue is as follows:

> 2. An info-gap analysis is not based on an estimate of the true horizon of uncertainty. That is, the info-gap model of uncertainty is **not** a single set, $U(\alpha, \tilde{u})$. Rather, an info-gap model is a family of nested sets, $U(\alpha, \tilde{u})$ for all $\alpha \geq 0$. The family of sets is usually unbounded[1]. Thus an info-gap model is not a "local analysis of risk" since the family of sets expands, usually boundlessly, as the unknown horizon of uncertainty, α, grows. Info-gap theory is **not** a worst-case analysis, since there is no known worst case in an info-gap model of uncertainty.
>
> Ben-Haim (2007, p. 2)
>
> [1] The family of sets is bounded only when there is a physical or definitional limit of the range of variation, such as probabilities not being larger than unity, or masses not being negative.

Thus, like most info-gap advocates, Ben-Haim (2007) confuses two facts about info-gap decision theory:

- The neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$ are allowed to expand the uncertainty set $\mathscr{U}$, so that as α grows, at the limit, $U(\infty, \tilde{u}) = \mathscr{U}$.
- Info-gap's robustness model [takes no notice whatsoever of] the performance of decision q outside the neighborhood $U(\alpha^*, \tilde{u})$, where $\alpha^* = \hat{\alpha}(q, r_c) + \varepsilon$ and ε can be arbitrarily small, but positive.

Incredible though it may sound, info-gap advocates are unclear on how these two facts relate to one another. What seems to be totally lost on them is that the neighborhoods $U(\alpha, \tilde{u}), \alpha \geq 0$ may very well expand boundlessly as α grows. The point remains though that info-gap's robustness analysis is driven by the constraint $r_c \leq r(q, u), \forall u \in U(\alpha, \tilde{u})$ on α to imply that the **admissible values** of α pertaining to decision q are restricted to the range $[0, \hat{\alpha}(q, r_c)]$. In other words, within the framework of info-gap's robustness analysis, the admissible value of α is **bounded above** by $\hat{\alpha}(q, r_c)$.

This means that info-gap's robustness model [takes not the slightest account] of the performance of decision q outside the neighborhood $U(\alpha^*, \tilde{u})$ which clearly indicates that this model is a model of local robustness.

Neither info-gap's robustness model, nor the users of the theory, have direct control of the value of α^*, and therefore the model admits cases where α^* is small. In particular, in the typical case, namely where $\mathscr{U}$ is unbounded, $U(\alpha^*, \tilde{u})$ would typically be extremely smaller than $\mathscr{U}$.

All this goes to show that info-gap's robustness model is a model of local robustness par excellence.

Interestingly, a contradictory position to that stated in Ben-Haim (2007) is stated in 2009 were we read the following:

> Sniedovich notes that the info-gap robustness function is "local" to the region around $\tilde{u}$, where $\tilde{u}$ is likely to be substantially in error. He concludes that therefore the info-gap robustness function is an unreliable assessment of immunity to error. There are several possible responses to this concern.
>
> ... ...
>
> ... ...
>
> Thus it is correct that the info-gap robustness function is local, with respect to $\tilde{u}$. However, the value judgment of whether this neighborhood of robustness is small, too small, large, large enough, etc., is characteristic of all decisions under uncertainty. A major purpose of quantitative decision analysis is to provide focus for the subjective judgments which must be made.
>
> Ben-Haim (2009)
> http://en.wikipedia.org/wiki/User:Ybenhaim/sandbox
> This page was last modified on 21 April 2009

The latest news is this:

> If the robustness is large (and this is a judgment that the analyst must make, like other judgments made by risk analysts) then one may have confidence in the decision. If the robustness is not large, and especially if the robustness is small, then confidence is not warranted. If the robustness is small then confidence is warranted only "locally," near the models, while if the robustness is large then confidence is warranted over a wide domain of deviation from the models. Info-gap theory uses the analyst's models, but this does not make it a "local" theory of robustness.
>
> Ben-Haim (2012a, p. 7)

For the benefit of readers who may find it difficult to decode the language of this argument, let me point out the following.

Info-gap's robustness model is a model of *local robustness,* hence info-gap decision theory is a theory of local robustness, not because it "uses the analyst's models". Rather, it is a theory of *local robustness* because its robustness model measures the robustness of decisions **in the first instance** against small perturbations in the value of $\tilde{u}$. In other words, it is a theory of *local* robustness because it completely ignores the performance of decision q outside the neighborhood $U(\alpha^{*}, \tilde{u})$. What is more, it makes no provisions for handling situations where $U(\alpha^{*}, \tilde{u})$ is much smaller than $\mathscr{U}$.

Let me elaborate on the "provision" issue.

According to info-gap decision theory, a small (indeed, the smallest) α^{*} suffices to determine that decision q is fragile. Once this determination is made, no attempt is made to evaluate the robustness of decisions q against variations in the value of u outside the neighborhood $U(\alpha^{*}, \tilde{u})$. This clearly emerges from the following statements:

> When $\hat{\alpha}$ is large the decision is stable with respect to the uncertainties. This means that the decision is steady over a wide range of values of the uncertainty variables u. On the other hand, if $\hat{\alpha}$ is small then the decision is fragile with respect to the uncertainty: small variation of u can cause the decision to change by at least r_c. If the $\hat{\alpha}$ is large the decision is robust, insensitive to uncertainties, and the algorithm can be reliably used for making the decision. If $\hat{\alpha}$ is small then the algorithm cannot be relied upon to yield

consistent decisions.

Ben-Haim (2006, p. 42)

> The ratio $\hat{\alpha}_e(q, r_c)/\tilde{c}(q)$ is a dimensionless expression of the greatest acceptable fractional uncertainty in $\tilde{c}(q)$. If $\hat{\alpha}_e(q, r_c)/\tilde{c}(q) \gg 1$ then large fractional variation of $c(q)$ with respect to the nominal function does not jeopardize attainment of the critical profit r_c, and the production volume q is very robust. On the other hand, if $\hat{\alpha}_e(q, r_c/\tilde{c}(q) \ll 1$, then even small fractional deviation of $c(q)$ from its nominal value entails shortfall below the critical profit, so this value of q is extremely fragile to uncertainty.

Ben-Haim (2006, p. 95)

That is, Ben-Haim's (2006, 2012a) own arguments confirm that info-gap decision theory is a theory of *local* robustness par excellence. This is brought out by the claim that, a decision is deemed fragile simply on grounds of its inability to withstand **a single small** perturbations/deviation in $\tilde{u}$.

Put another way, that the evaluation of a decision q is *local* is broadcast loud and clear by the fact that the smallest perturbations/deviations in $\tilde{u}$, suffices to deem the decision fragile, END OF STORY. This determination is made regardless of the decision's behavior elsewhere on the uncertainty space, as no attempt whatsoever is made to check how this decision fares against large deviations/perturbations in $\tilde{u}$.

The whole point is that, in general, the fact that decision q cannot withstand small perturbations/deviations in $\tilde{u}$ **is no indication** that it is not resilient to large perturbations/deviations (see Figure 12, Figure 13).

And so, a decision q may well be judged fragile by info-gap decision theory, regardless of its resilience to $\boxed{\text{all larger}}$ deviations/perturbations.

That said, it is important to have a good grasp of info-gap's stipulation that as far as robustness is concerned, the larger $\hat{\alpha}(q, r_c)$ the better. Consider then the following statements:

> A cautious decision maker desires immunity to uncertainty and prefers a large rather than a small value of $\hat{\alpha}$: hence the preference ordering of relation (3.171).

Ben-Haim (2006, p. 102)

> The robustness function $\hat{\alpha}(q, r_c)$ is the immunity against failure: the greatest level of uncertainty which is consistent with always attaining no less that a specific critical level of reward. When $\hat{\alpha}(q, r_c)$ is large then great ambient uncertainty can be tolerated, which is clearly desirable. "Bigger is better" as far as $\hat{\alpha}(q, r_c)$ is concerned.

Ben-Haim (2006, p. 129)

> When operating under severe uncertainty, a decision which is guaranteed to achieve an acceptable outcome throughout a large range of uncertain realizations is preferred to a decision which can fail to achieve an acceptable outcome even under small error. In this way the robustness function generates preferences among available decisions. When choosing between two options, the *robust-satisficing* decision strategy selects the more robust option.

Ben-Haim (2010, p. 8)

It is important to realize that the argument that: if the info-gap robustness of decision q is large, then "confidence is warranted over a wide domain of deviation from the models" **does not rule out the fact** that the robustness of decision q is *local*. The implication being that there may still be another decision warranting greater confidence because, large robustness determined *locally* in not the end of the story!

To see that this is so, consider two decisions, say q' and q'', whose info-gap robustness is very large and $\hat{\alpha}(q', r_c)$ is slightly smaller than $\hat{\alpha}(q'', r_c)$. Info-gap decision theory rules that q'' is more robust than q'. But the point to note here is that decision q' can still outperform decision q'' outside the set $U(\hat{\alpha}(q'', r_c), \tilde{u})$. Hence, when the performance of these decisions is evaluated over the entire uncertainty space $\mathscr{U}$, decision q' would turn out to be more robust than decision q''.

In short, the argument in Ben-Haim (2012a, p. 7) actually corroborates the fact that info-gap's robustness model is a model of local robustness par excellence. The impression one has is that the errors in this as well as in other arguments in Ben-Haim's (2006, 2012) stem from the erroneous assumption that a decision q that cannot withstand small perforations/deviations in $\tilde{u}$, cannot withstand larger deviations. Obviously, this assumption might be true for some trivial cases, but it definitely does not hold in general.

Finally, I want to examine briefly a new myth about the prowess of info-gap decision theory that surfaced recently in an article published in *Risk Analysis*. It goes like this:

> For small α, searching set $U(\alpha, \tilde{u})$ resembles a local robustness analysis. However, α is allowed to increase so that in the limit the set $U(\alpha, \tilde{u})$ covers the entire parameter space and the analysis becomes one of global robustness. The analysis of a continuum of uncertainty from local to global is one of the novel ways in which info-gap analysis is informative.
>
> Hall et al. (2012, p. 6)

As I devote a whole article to debunk this fallacy (See Sniedovich 2012c), I shall not elaborate on it here. For our purposes it suffices to point out that info-gap's robustness model does not allow the analyst to vary the range of admissible values of α, as mistakenly claimed by Hall et al. (2012). The procedure that Hall et al. (2012) refer to has to do with the construction of the info-gap *robustness curve* for decision q, which shows the variation in the value of $\hat{\alpha}(q, r_c)$ relative to the variation in the value of r_c. But, this procedure $\boxed{\text{does not}}$ yield the global robustness of decision q with respect to $r_c \leq r(q, u)$ for the given value of r_c under consideration.

The only exception to this fact is the inconsequential case where r_c is so small that $\hat{\alpha}(q, r_c)$ is equal to ∞. But, the point is that here $r_c \leq r(q, u), \forall u \in \mathscr{U}$, which means that the constraint $r_c \leq r(q, u)$ in fact turns out to be $\boxed{\text{redundant.}}$ This means that the procedure suggested by Hall et al. (2012) to generate the global robustness of decision q is a $\boxed{\text{lamppost type procedure}}$: search for your lost keys under the nearest lamppost, not in the dark alley where they were actually lost.

Let me explain.

Suppose that we need to determine the $\boxed{\text{global}}$ robustness of decision q with respect to the constraint $\boxed{348} \leq r(q, u)$ in a situation where

$$r(q, \mathscr{U}) := \{r(q, u) : u \in \mathscr{U}\} = [2, 1000]. \tag{41}$$

Info-gap decision theory does not have the capabilities do this. All it can do is to determine the $\boxed{\text{local}}$ robustness of decision q with respect to $\boxed{348} \leq r(q, u)$ by means of its robustness model:

$$\hat{\alpha}(q, 348) := \max_{\alpha \geq 0} \{\alpha : 348 \leq r(q, u), \forall u \in U(\alpha, \tilde{u})\}. \tag{42}$$

But, this is not what we are looking for. We want to determine the $\boxed{\text{global}}$ robustness of decision q with respect to $\boxed{348} \leq r(q, u)$.

So, if we adopt Hall et al's. (2012) proposition that "in the limit the set $U(\alpha, \tilde{u})$ covers the entire parameter space the analysis becomes one of global robustness", all we need to do is to establish the $\boxed{\text{global}}$ robustness of decision q with respect to the constraint $\boxed{2} \leq r(q, u)$.

The point is, however, that this constraint turns out to be $\boxed{\text{redundant,}}$ so what we have here is the following:

$$\begin{aligned}\hat{\alpha}(q, 2) :&= \max_{\alpha \geq 0} \{\alpha : 2 \leq r(q, u), \forall u \in U(\alpha, \tilde{u})\} && (43)\\ &= \infty. && (44)\end{aligned}$$

This means that we can now confidently claim the following:

- Decision q is super-robust on $\mathscr{U}$ with respect to the constraint $2 \leq r(q,u)$. In other words, $2 \leq r(q,u), \forall u \in \mathscr{U}$, observing that this robustness analysis is global.

The trouble with this "novel and informative" approach is that:

- To obtain this obvious "result" we do not have to turn to info-gap decision theory . Indeed, using info-gap's robustness analysis for this purpose is utterly counter-productive.
- This result is not a solution to the task at hand. Our task is to determine the global robustness of decision q with respect to the constraint $348 \leq r(q,u)$.

A full discussion on in Hall et al.'s (2012) misrepresentation of info-gap decision theory can be found in Sniedovich (2012c).

10.6 Widest range of contingencies

Strange though it may sound, info-gap scholars, the Father of this theory included, appear to confuse info-gap robustness with *Global Size-Robustness.* Put another way, info-gap scholars, the Father of this theory included, ascribe to info-gap's robustness model the capabilities of the *Global Size-Robustness* model.

Since the difference between the *info-gap robustness* of a decision and the *Global Size-Robustness* of a decision is clear as daylight, it is hard to imagine how these two measures of robustness with respect to the performance constraint $r_c \leq r(q,u)$ can be confused at all, for consider the following comparison:

Info-gap robustness	Global Size Robustness
$\max_{\alpha \geq 0} \{\alpha : r_c \leq r(q,u), \forall u \in U(\alpha,\tilde{u})\}$	$\max_{V \subset \mathscr{U}} \{size(V) : r_c \leq r(q,u), \forall u \in V\}$

In words,

Info-gap robustness	Global Size Robustness
The size, α, of the largest neighborhood, $U(\alpha,\tilde{u})$, around $\tilde{u}$, all of whose elements satisfy the performance constraint $r_c \leq r(q,u)$.	The size of the largest subset V of the uncertainty space $\mathscr{U}$ all of whose elements satisfy the performance constraint $r_c \leq r(q,u)$.

To see what I have in mind, consider the following statements:

> Info-gap analysis allows the decision maker to identify solutions that perform satisfactorily well under the widest possible range of conditions.
>
> Hall and Ben-Haim (2007, p. 7)

> When operating under severe uncertainty, a decision which is guaranteed to achieve an acceptable outcome throughout a large range of uncertain realizations is preferred to a decision which can fail to achieve an acceptable outcome even under small error.
>
> Ben-Haim (2010, p. 8)

> The maximizer of utility seeks the answer to a single question: which option provides the highest subjective expected utility. The robust satisficer answers two questions: first, what will be a "good enough" or satisfactory outcome; and second, of the options that will produce a good enough outcome, which one will do so under the widest range of possible future states of the world.
>
> Schwartz Ben-Haim and Dasco (2011, p. 213)

> It asks, instead, "What kind of return do we want in the coming year, say, in order to compare favorably with the competition? And what strategy will get us that return under the widest array of circumstances?"
>
> Schwartz, Ben-Haim and Dasco (2011, p. 220)

> For an individual who recognizes the costliness of decision making, and who identifies adequate (as opposed to extreme) gains that must be attained, a satisficing approach will achieve those gains for the widest range of contingencies.
>
> Schwartz, Ben-Haim and Dasco (2011, p. 223)

> Decisions that cause the system to exceed the performance criterion over a wide range of uncertainty are said to be more "robust" or "immune to failure" (Ben-Haim 2006).
>
> van der Burg and Tyre (2011, 304)

One imagines that this rhetoric, which utterly misrepresents the facts about info-gap's robustness model, hence info-gap robustness, is an inevitable consequence of the misconceptions surrounding the great fuss that is made in the info-gap literature regarding info-gap's ability to deal with an extreme uncertainty that is manifested in an unbounded uncertainty space. Because, if info-gap decision theory is trumpeted to have the capabilities to handle an unbounded uncertainty space, then the next step would be to attribute to its robustness analysis the capability to determine the widest range of acceptable outcome in this space.

The truth is of course, that this characterization of info-gap robustness flies in the face of info-gap's very definition of robustness.

The important point to note about these statements is that their characterization of info-gap robustness is completely oblivious to the central role that the point estimate $\tilde{u}$ plays in an info-gap robustness analysis, and which therefore renders the analysis and its results *inherently local.* This means of course that info-gap's robustness analysis does not identify solutions that perform satisfactorily well under the widest possible range of conditions. Rather, it seeks a decision that performs satisfactorily over the largest *neighborhood* $U(\alpha,\tilde{u})$ *around the nominal point* $\tilde{u}$..

11 Discussion

My objective in this article was to explain in technical terms and to illustrate graphically what makes a decision theory a *voodoo decision theory.*

The main thesis of this article is then that the hallmark of such a theory is the proposition to employ a radius of stability model to determine the robustness of decisions against an *extreme uncertainty* of the type postulated by info-gap decision theory (IGT). Such a theory effectively proposes an analysis in domain $\mathscr{D}$, which typically is a minutely small neighborhood in the vast uncertainty space $\mathscr{U}$, as a reliable method for dealing with such an extreme uncertainty. This, as I show here, in fact comes down to a proposition to dodge all the difficulties and challenges posed by *extreme uncertainty.*

To the best of my knowledge, info-gap decision theory is the only decision theory to make this proposition.

For a slightly different perspective on this point, consider the following statement:

> Analysts who were attracted to IGT because they are very uncertain, and hence reluctant to specify a probability distribution for a model's parameters, may be disappointed to find that they need to specify the plausibility of possible parameter values in order to identify a robust management strategy.
>
> Hayes (2011, p. 88)

Despite its cautious language, this statement manages to convey, albeit rather diplomatically, that IGT actually proposes to engage in ALCHEMY.

Because, what this statement in fact intimates is that while a big fuss is made, in the info-gap literature, about info-gap's model of uncertainty being probability-free, likelihood-free, plausibility-free, info-gap's prescription for the management of uncertainty flies in the face of this fundamental stipulation.

Astute analysts therefore realize that one cannot square the instruction to employ a model of uncertainty that is claimed to be probability-free, likelihood-free, plausibility-free, and so on, with the instruction to designate a "wild guess" as the fulcrum of an analysis seeking to identify a robust management strategy. To this I should add that neither would astute analysts expect an analysis that is confined to domain $\mathscr{D}$, which typically is a minutely small neighborhood in the vast uncertainty space $\mathscr{U}$ to identify a robust management strategy.

The point is, however, that these facts continue to evade many analysts, referees of peer-reviewed journals included, because of the heavy **rhetoric** in which they are dressed. Indeed, my experience has shown that it takes a considerable amount of expertise, in a number of areas (e.g. robust optimization, control theory, statistics et.) to sort out the facts from the **rhetoric** describing the theory, its robustness model and the results yielded by this model and so on.

And the trouble is that not only do the **rhetoric** about info-gap's robustness model its mode of operation, capabilities and scope mislead referees of peer-reviewed journals. They seem to entrench absurd ideas, in info-gap circles, about the capabilities of info-gap's robustness model, such as its being a reliable tool for the management of a severe uncertainty that is manifested in extreme events such as seawalls, massive tsunamis, catastrophes and so on and even ... *Black Swans* and *Unknown unknowns* (Wintle et al. 2010).

For an excellent illustration of where info-gap's misleading rhetoric might land you, consider Sims' (2001) warning in *Pitfalls of a Minimax Approach to Model Uncertainty* about an unguarded use of minimax models of $\boxed{\text{local}}$ robustness in macroeconomics:

> They may also—and this is more likely in the recent implementations in macroeconomics—focus the minimaxing on a narrow, technically convenient, uncontroversial range of deviations from a central model. Then the results will remain close to those of the central model, and the danger is that one will be misled by the rhetoric of robustness into devoting less attention than one should to technically inconvenient, controversial deviations from the central model.
>
> Sims (2001, p. 52)

Another illustration of the consequences of being misled by the rhetoric on info-gap's robustness model is the danger of using models that befit, what Ben-Tal et al. (2009a, p. 926) term *'... somewhat "irresponsible" decision makers... '*, namely decision makers who confine their robustness analysis to the "normal" range of values of the uncertainty parameter thus ignoring "abnormal" values of this parameter.

I am not familiar with another theory of local robustness whose rhetoric is as misleading as that of info-gap decisions theory.

12 Conclusions

As explained in this discussion, one does not have to be a risk analyst to appreciate that info-gap decision theory not only does not deal with the difficulties presented by a severe uncertainty of the type that it claims to address, but that it dodges these difficulties altogether.

Indeed, one does not have to be a risk analyst to figure out that a theory that is based on a local robustness analysis of the radius of stability type, lacks the capabilities that are required to tackle the fundamental technical and methodological difficulties that are presented by an extreme uncertainty that is characterized by a vast uncertainty space, a poor point estimate, and a likelihood-free quantification of uncertainty.

What is the wonder then, that other than info-gap decision theory, no other decision theory proposes radius of stability models to tackle an extreme uncertainty of this type. Of course, radius of stability models have, for decades, been used extensively in many disciplines for the management of small perturbations in a nominal value of a parameter and they continue to provide excellent, reliable tools for this purpose. But their local orientation renders them utterly unsuitable for the treatment of uncertainties that are as severe as those stipulated by info-gap decision theory.

No amount of rhetoric can change this fact, much less can rhetoric take on the challenges posed by extreme uncertainty. If anything, as shown by the info-gap experience, rhetoric will only serve to slow down progress in this important area. And to illustrate, as I explain in the article *Robust optimization: the elephant in the robust-satisficing room,* the rhetoric about the robust-satisficing approach promoted by info-gap may well mislead its followers back to the 1960s, namely to the early days of robust optimization.

Finally, no amount of rhetoric can change the fact that info-gap's robustness model is a re-invented version of the radius of stability model, hence a simple instance of Wald's maximin model. Readers who remain unclear as to why info-gap's robustness model is a model of local robustness should reflect on the implications of Figure 16.

So, all that is left to say is that it is most unfortunate that peer-reviewed journals, such as *Risk Analysis,* continue to provide a platform for a fundamentally flawed theory such as info-gap decision theory. One would have expected referees of journals devoted to the study of risk, such as *Risk Analysis,* to be more vigilant about the claims regarding the nature of this theory, and the its place in the state of the art.

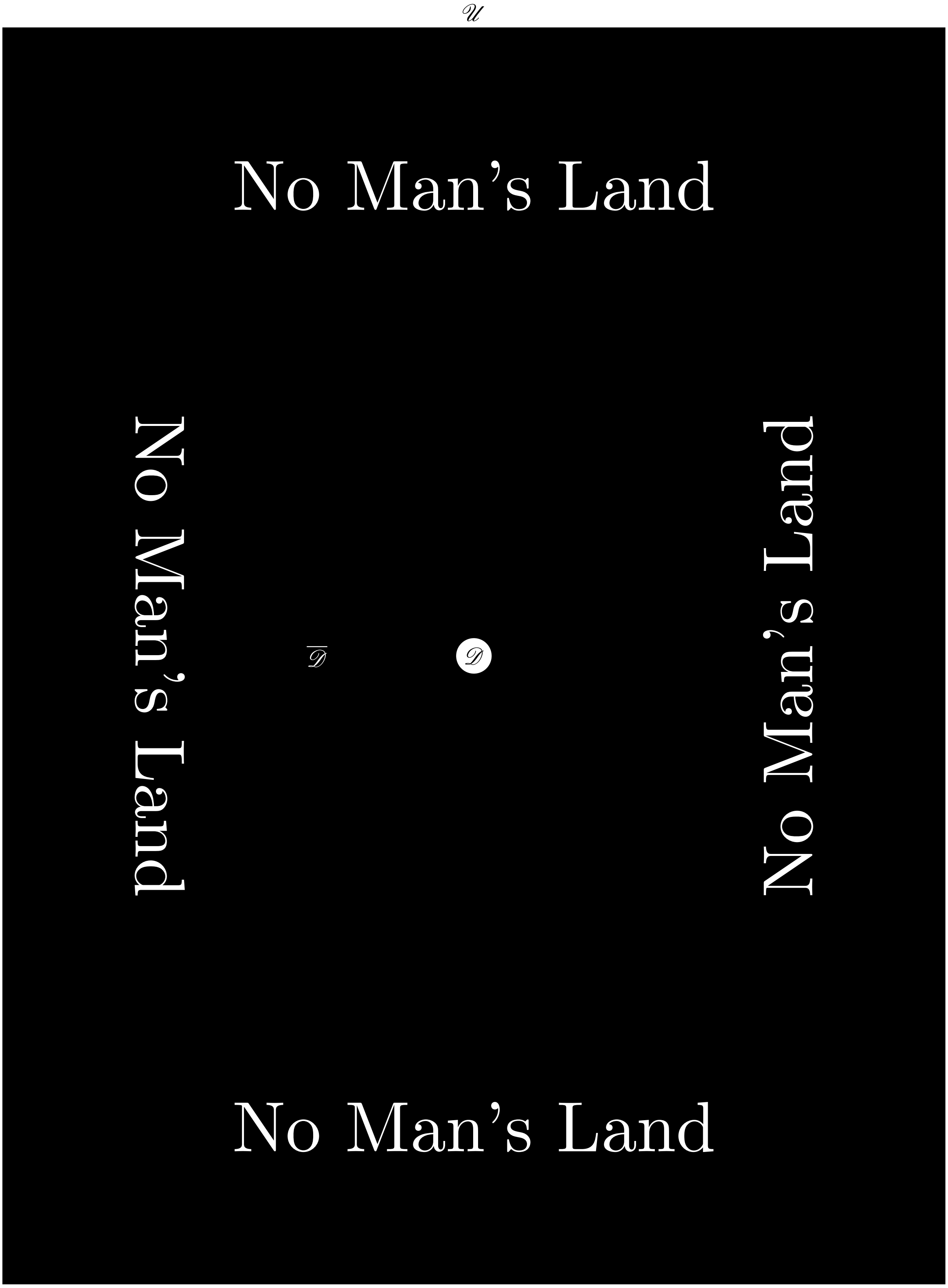

Figure 16: A profile of info-gap's robustness analysis. The uncertainty space $\mathscr{U}$ is vast, and the domain of analysis, $\mathscr{D}$, is a small neighborhood around a wild guess of the true value of the parameter of interest.

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