

# The Art, Science, and Joy of (**mathematical**) Decision-Making

Moshe Sniedovich

Department of Mathematics and Statistics  
The University of Melbourne  
[www.moshe-online.com](http://www.moshe-online.com)

Black Box Lecture Series

June 20, 2012  
DSTO, Melbourne

# Black Box

<http://www.merriam-webster.com/dictionary/black+box>

## **black box** noun

1: a usually complicated electronic device whose internal mechanism is usually hidden from or mysterious to the user; broadly : anything that has mysterious or unknown internal functions or mechanisms.

2 : a crashworthy device in aircraft for recording cockpit conversations and flight data

First known use: circa 1945.

# Admin

This is a

Math Classification G

presentation.

Math Classification MA +18

versions can be found at

[decision-making.moshe-online.com](http://decision-making.moshe-online.com)

Math-oriented lectures/workshops on this and related topics  
can be easily arranged.

## Public Notice

- There is not need to take notes.

A copy of this presentation is available on my website at

<http://decision-making.moshe-online.com/ballarath.html>

- I regret to advise that the short exam at the end of the lecture was canceled.
- So, relax and enjoy the presentation!



# The Plan

- Start with a math-free discussion
- Increase the mathematical content gradually
- Keep it simple!

The plan will fail if

- I'll lose everyone; or
- I'll lose no one

before the end of the lecture.

FYI: There are only 3 (distinct) math expressions in this presentation!

## About the title

# The Art, Science, and Joy of (mathematical) Decision-Making

### Nature and Mathematics

The laws of Nature are written in the language of mathematics.

Galileo Galilei (1564-1642)

Italian physicist, mathematician, astronomer, and philosopher

### Science and Mathematics

Mathematics is the Queen of Science, and Arithmetic the Queen of Mathematics.

Johann Carl Friedrich Gauss (1777-1855)

German mathematician and scientist

## About the title

# The Art, Science, and Joy of (mathematical) Decision-Making

### The Unreasonable Effectiveness of Mathematics

How can it be that simple mathematics, being after all a product of the human mind, can be so remarkably useful in so many widely different situations?

R. W. Hamming

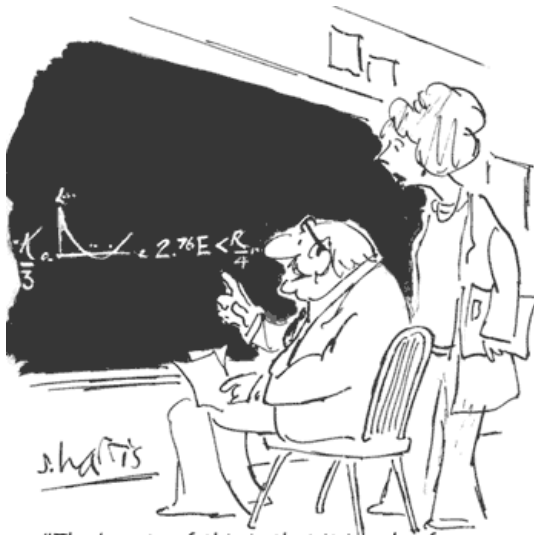
The American Mathematical Monthly

Volume 87 Number 2, 1980

### Partial Answer

The relationship between mathematics and other disciplines is not unidirectional: it is **synergetic**.

## About the title



*"The beauty of this is that it is only of theoretical importance, and there is no way it can be of any practical use whatsoever."*

## About the title

# The Art, Science, and Joy of (mathematical) Decision-Making

### Bottom line

There is a need to make decision-makers more aware of the important role that mathematics can play in decision-making.



® <http://maddmaths.simai.eu/leditoriale/la-matematica-dentro>

# Decision-Making

This topic can be discussed from a variety of viewpoints, e.g.:

- **Sociology** of decision-making
- **Psychology** of decision-making
- **Behavioral** aspects of decision-making
- **Politics** of decision-making
- **Mathematics of decision-making**

## Difficulty

How do you do the latter in the framework of a **math-free** public lecture?!?

You'll see many ... pictures and be given many pointers!)

# Decision-Making

## Acknowledgment

Most of the following quotes are from

[http://www.decision-making-solutions.com/  
decision\\_making\\_quotes.html](http://www.decision-making-solutions.com/decision_making_quotes.html)



# Decision-Making-Solutions

You may wish to read them on you own after the lecture.

## Decision-Making

### Mark Twain (1835 - 1910), Novelist and Journalist

Good decisions come from experience. Experience comes from making bad decisions.

### Thomas Hardy (1840 - 1928), Novelist and Poet

The offhand decision of some commonplace mind high in office at a critical moment influences the course of events for a hundred years.

### Albert Camus (1913 - 1960), French Author

Life is the sum of all your choices.

### Jason Seiden, American Author

Take ambiguity away from leadership, and you take away tough decisions and responsibility. What you're left with is overpaid administration.



## Decision-Making

### **Napoleon Bonaparte (1769 - 1821), Leader of France**

Nothing is more difficult, and therefore more precious, than to be able to decide.

### **Theodore Roosevelt (1854 - 1919), President of the US**

In any moment of decision the best thing you can do is the right thing, the next best thing is the wrong thing, and the worst thing you can do is nothing.

### **Lyndon B. Johnson (1908 - 1973), President of the US**

Presidents quickly realize that while a single act might destroy the world they live in, no one single decision can make life suddenly better or can turn history around for the good.

## Decision-Making

### **Bob U. Woodward (1943 - ), Investigative Reporter**

When you see how the President makes political or policy decisions, you see who he is. The essence of the Presidency is decision-making.

### **Martin Luther King, Jr. (1929 -1968), Clergyman**

Man is man because he is free to operate within the framework of his destiny. He is free to deliberate, to make decisions, and to choose between alternatives.

### **1st Earl of Mansfield (1705 - 1793), Politician**

Consider what you think justice requires, and decide accordingly. But never give your reasons; for your judgment will probably be right, but your reasons will certainly be wrong.

## Decision-Making

### William James (1842 - 1910), American Philosopher

When you have to make a choice and don't make it, that is in itself a choice.

### Unknown

Indecision becomes decision with time.

### Pythagoras (570 BC - 495 BC), Greek philosopher

Choices are the hinges of destiny.

### Kevin Kelly (1967 - ), American Professional Boxer

And they discovered something very interesting: when it comes to walking, most of the ant's thinking and decision-making is not in its brain at all. It's distributed. It's in its legs.

## Decision-Making

### Optimum Optimorum Principle

There comes a time when one must stop suggesting and evaluating new solutions, and get on with the job of analyzing and finally implementing one pretty good solution.

### William Edwards Deming (1900 - 1993), Statistician

The ultimate purpose of collecting the data is to provide a basis for action or a recommendation.

### Isaac Asimov (1920 - 1992), Science Fiction Author

It is change, continuing change, inevitable change, that is the dominant factor in society today. No sensible decision can be made any longer without taking into account not only the world as it is, but the world as it will be.

## Decision-Making

### Warning/advice

Beware of

- books
- presentations
- videos
- software
- experts
- management consultants
- financial advisors
- neighbors
- relatives

that promise simple, foolproof recipes for “great decisions”.  
There ain't no such a magic recipe!

## Examples of decision-making problems

- Shortest path
- Airline operations
- Mining operations
- Portfolio investment
- Bio and homeland security
- Health-care
- Climate change
- Defense/military
- Communication
- Entertainment
- Supply chain
- Online auctions
- Voting systems

## Examples of decision-making problems

What makes a “problem” a **decision-making problem**?

### Elements of a decision-making problem

- Decisions variables (“unknowns”, to be determined)
- Objectives and goals
- Requirements and constraints
- Preference structure
- Parameters

Think about these elements as we discuss the following examples.

## Shortest path

<http://www.travelmate.com.au>

Find your way around Australia.  
Map Maker will give you maps, driving directions and estimated driving times.

From: VIC ▾ Melbourne

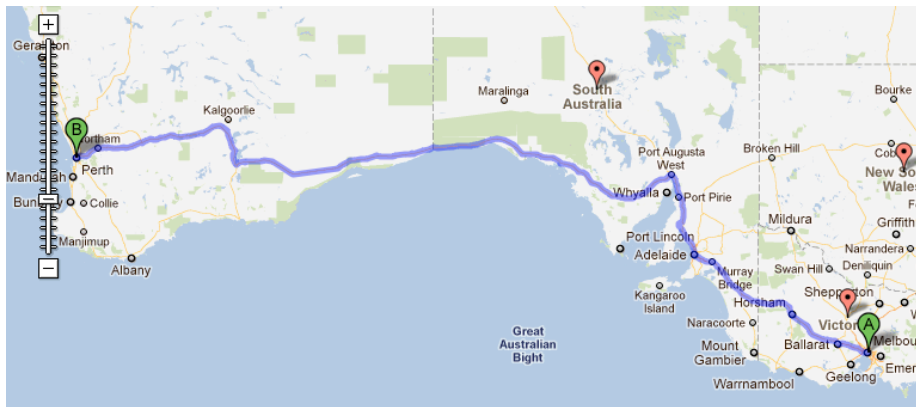
To: WA ▾ Perth

go



## Shortest path

<http://maps.travelmate.com.au>



Note: the same (math) model that represents shortest path problems also represents numerous other (quite different) decision-making problems.

# Airline operations



\$200,000,000



35,000 employees

Expensive operation (\$16,200,000,000 annual turnover):

- Crew/flight scheduling (5,580 domestic flights a week)
- Equipment replacement (300 planes)

Qantas is the oldest continuously operated airline in the world (since November 16, 1920) and the second oldest overall.

## Online auctions



[http://en.wikipedia.org/wiki/Paul\\_Milgrom](http://en.wikipedia.org/wiki/Paul_Milgrom)

### Professional life

Milgrom has done a great job as a consultant. In his most recent company, Auctionomics, he had advised many big corporations and helped them save money in big stake auctions. In one auction, he consulted Time Warner and Comcast, and they paid about a third less than their competitors for equivalent spectrum, saving almost \$1.2 billion.

# Online auctions

<http://www.auctionomics.com>



[Home](#)   [About](#)   [PRODUCTS](#)   [ADVISORY](#)   [PARTNERS](#)   [CLIENTS](#)   [IN THE PRESS](#)   [CONTACT US](#)

## WELCOME to AUCTIONOMICS



Auctionomics is an auction-design and software firm offering innovative, economically sound solutions to complicated problems. Our advanced online software [products](#) and cutting-edge auction designs give Auctionomics an unprecedented and unparalleled ability to design efficient markets for trading multiple goods, maximizing the gains from trade and creating win-win solutions for our clients, whether buying, selling, or swapping.

*... Our advanced online **soft**   **products** and  
 cutting-edge auction **design***



# Supply chain



1:30-2:30 PM

## Selling shovels to miners: Building the commercial space supply chain

Sponsored by United Launch Alliance

While miners were searching for gold, the people who sold **shovels to miners got rich**. A robust commercial space supply chain will provide stability, share risk for product development, create platforms for collaboration and enable long-term sustainability. This session will examine the benefits of supporting supply chain development. An organized supply chain will enable the formation of powerful and profitable alliances to develop and manufacture the products necessary for industry success.

## Supply chain

### Race to Mine the Moon Heats Up

By Loren Grush, Published October 27, 2011 | FoxNews.com



Astrobotic Technology's Red Rover, a lunar exploration vehicle that the company claims will be able to scout and drill for precious resources at the moon's poles.

# Mining operations

<http://www.superpit.com.au/>

## Welcome to the Super Pit

Based in Kalgoorlie, Western Australia, The Super Pit produces up to 850,000 ounces of gold every year and its operation far outweighs any other mining centre in Australia. The Super Pit is the biggest gold open pit mine in the country.



# Mining operations





# Mining operations

## Caterpillar 797



## Mining operations



The truck uses fuel in huge amounts ... an average of 65 gallons/hr ... with a fuel economy rating of 0.3 mpg. With such huge costs involved, the vehicle is usually run 24 hours per day, 365 days per year, stopping only for regularly scheduled maintenance.

<http://www.worsleyschool.net/science/files/extreme/cat797.html>

## Mining operations



Michelin tires for Caterpillar 797

\$42,500 each

Truly the most expensive tires in the world, the Michelin 59/80R63 XDR tires required for the Caterpillar 797 dump trucks are a massive 13 feet tall and weigh 11,680 lbs. They are the largest tires in the world. These tires require 47 nuts to be attached to the axle. Six of these enormous tires can hold up to 1,375,000 lbs of truck and load.

## Bio and homeland security



## A Special Example

### Remark

The next example is included in the discussion to stress that decision-theory is not a “capitalistic” theory.

# Voting system

## Task

Design a voting system that satisfies the following conditions:

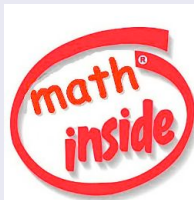
- **Individual Sovereignty:** individual should be able to order the alternatives in any way (including ties).
- **Unanimity:** If every individual prefers one alternative to another, then the group ranking should do the same.
- **Independence of Irrelevant Alternatives:** removal of an alternative does not affect to ordering of the remaining alternatives.
- **Uniqueness:** The system should yield the same ranking regardless of the order in which alternatives are compared.
- **Nondictatorship:** The preferences of an individual should not become the group ranking without considering the preferences of others.

## Voting system

### Arrow's Impossibility Theorem (circa 1951)

It is impossible to formulate a preference (voting) system that satisfies all the above conditions.

### Proof



Hint:

Prove that any system that satisfies the first four conditions violates the last, namely any such system is a ... dictatorship!

# Decision-Making

## Timeout # 1

The objective of these exmples is to point out that decision-making problems of the kind we consider here cannot be handled by soft, general purpose “guidelines” for great decision-making.



# Decision-Making

## Typical soft recipe for great decisions

### Nine Ways To Make Great Decisions

- 1 Define the issue.
- 2 Take a positive approach.
- 3 List your options.
- 4 Gather information about your options.
- 5 Be objective.
- 6 Consider your options.
- 7 Be true to yourself.
- 8 Make a decision.
- 9 Be open to change if circumstances change.

<http://finance.groups.yahoo.com/group/PassionHR/message/6908>

# About computers




## Observation

Solving decision-making problems by **complete enumeration** is not a viable option in the context of the problems under consideration in this discussion.

We need both **computing power** and **brain power**, including **math-power**.

# On the **mathematics** of decision-making

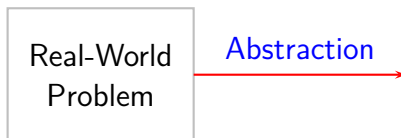
## Conceptual Framework



Real-World  
Problem

# On the **mathematics** of decision-making

## Conceptual Framework



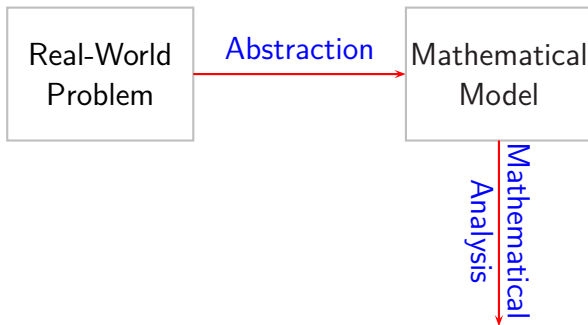
# On the **mathematics** of decision-making

## Conceptual Framework



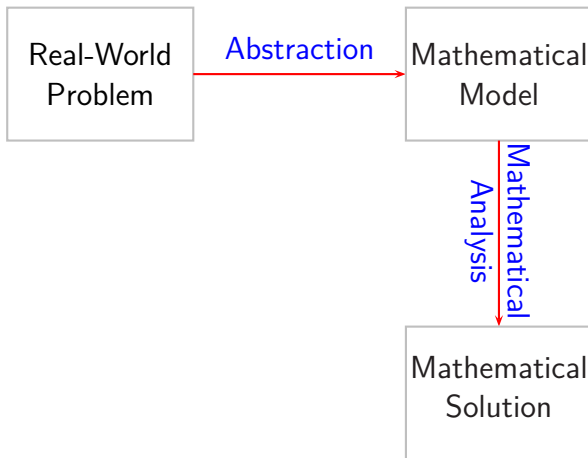
# On the **mathematics** of decision-making

## Conceptual Framework



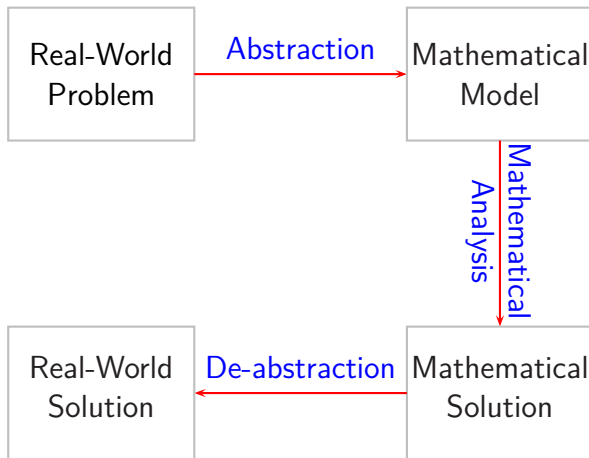
# On the **mathematics** of decision-making

## Conceptual Framework



# On the **mathematics** of decision-making

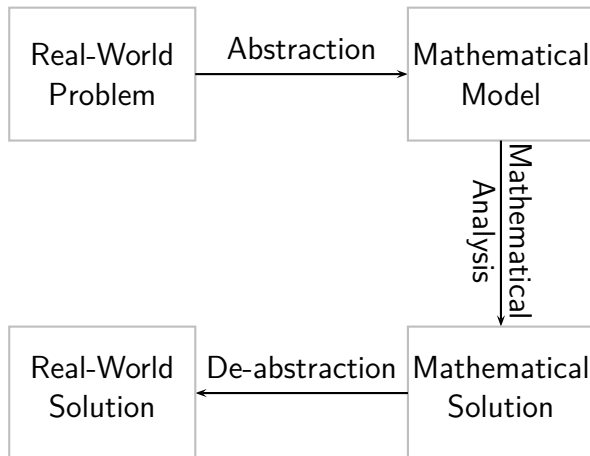
## Conceptual Framework





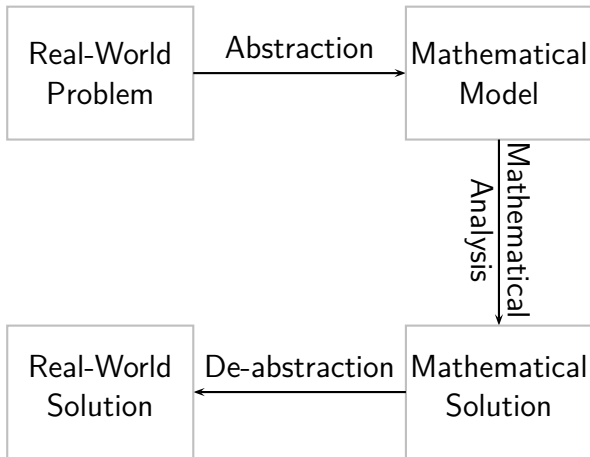
# On the **mathematics** of decision-making

## Conceptual Framework



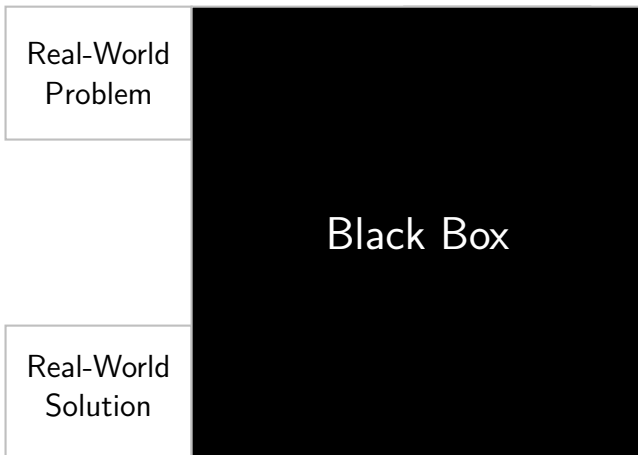
# On the mathematics of decision-making

## Marketing



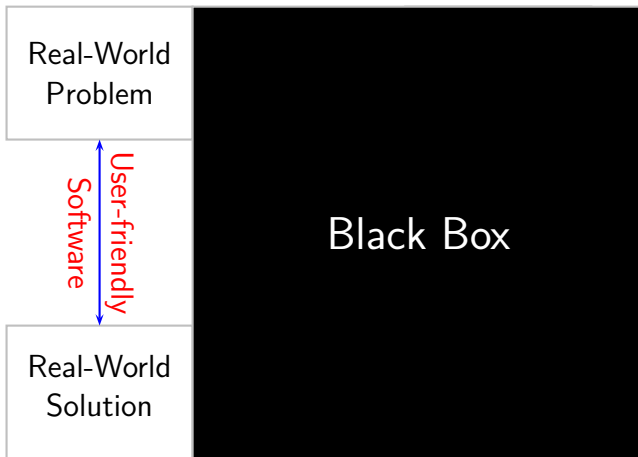
# On the mathematics of decision-making

## Marketing



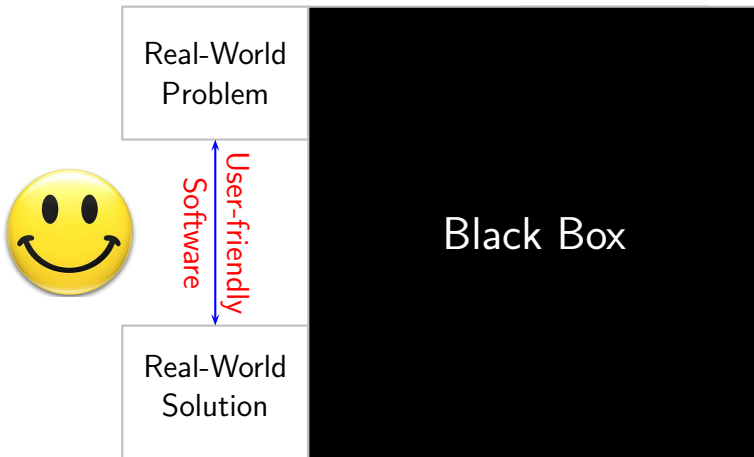
# On the mathematics of decision-making

## Marketing



# On the mathematics of decision-making

## Marketing

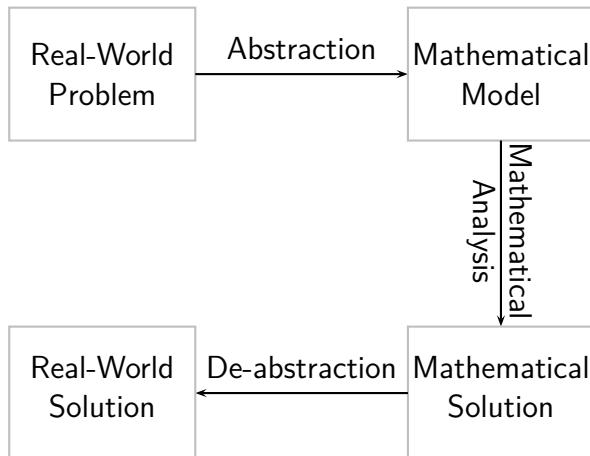


## On the mathematics of decision-making

Full story

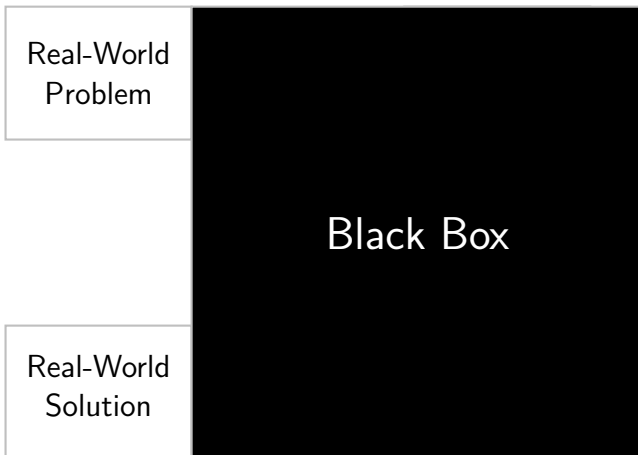
# On the mathematics of decision-making

## Full Story



# On the mathematics of decision-making

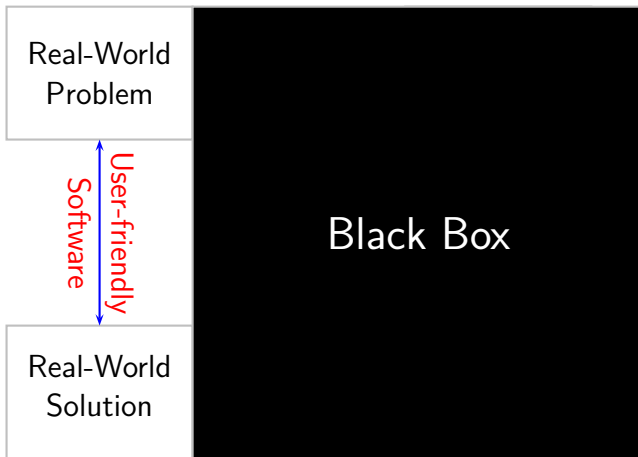
## Fyll Story





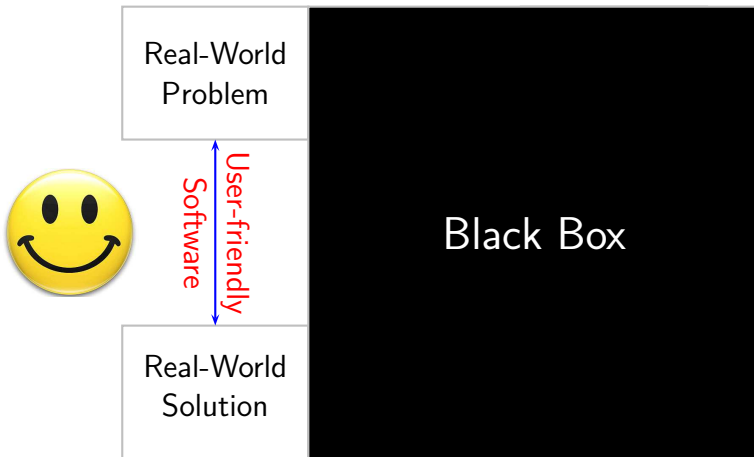
# On the mathematics of decision-making

## Full Story



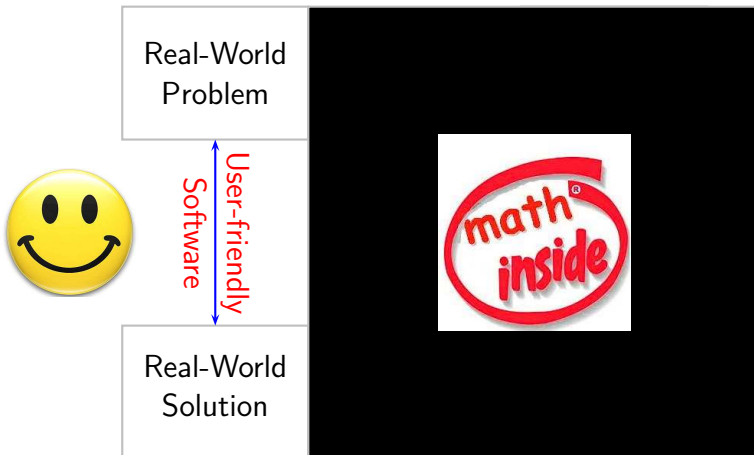
# On the mathematics of decision-making

## Full Story



# On the mathematics of decision-making

## Full Story



## Example: The Art of Math Modeling

### Public Notice / Apology

The following example may be classified by some as **politically incorrect**.

I have been using it for many years, well before it was classified as such.

## Example: The Art of Math Modeling

" ... Questions of taste were soon decided in those days. When a twelfth-century youth fell in love, he did not take three paces backward, gaze into her eyes and tell her she was beautiful to live. He said he would step outside and see about it. And if, when he got out, he met a man and broke his head – the other's man's head, I mean – then that proved that his – the first fellow's girl – was a pretty girl. But if the other fellow's – the other fellow to the second fellow, that is because of course the other fellow would only be the other fellow to him, not the first fellow, who – well, if he broke his head, then his girl – not the other fellow's, but the fellow who was the – Look here, if **A** broke **B**'s head, then **A**'s girl was a pretty girl, but if **B** broke **A**'s head, then **A**'s girl wasn't pretty girl, but **B**'s girl was. That was their method of conducting art criticism. ... "

## Example: The Art of Math Modeling

“... Now-a-days we light a pipe, and let the girls fight it out amongst themselves ...”

Jerome K. Jerome  
 Idle Thoughts of an Idle Man,  
 Being Idle, pp. 58-59, 1889.

## Example: The Art of Shoe Lacing



Vincent Van Gogh: Old Shoes, July-September 1886

*"... Van Gogh often painted shoes, especially old shoes, which to him symbolized the whole universe of the wearer – his pains, sorrows, hard labor, tiredness at the end of the day ..."*

<http://www.shoeblog.com/blog/friday-shoe-history-corner-van-gogh-paints-shoes/>

## Example: The Art of Shoe Lacing

### Ian's Shoelacing Site

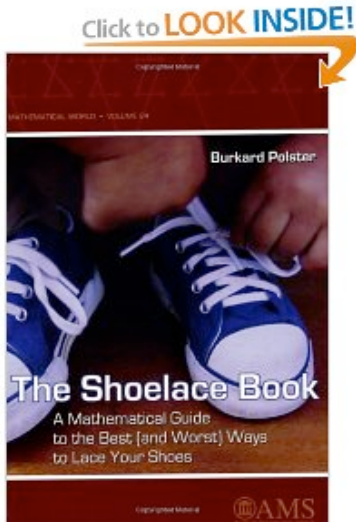


*"... Whilst mathematics tells us that there are more than **2 Trillion ways** of feeding a lace through the **six pairs** of eyelets on an average shoe, this section presents a fairly extensive selection of **36 shoe lacing tutorials**. They include traditional and alternative lacing methods that are either widely used, have a particular feature or benefit, or that I just like the look of..."*

<http://www.fieggen.com/shoelace/lacingmethods.htm>



## Example: The Art of Shoe Lacing



# The Art of Shoe Lacing

## Remark

This decision-making problem is a very simple instance of an extremely important generic decision-making problem ...

## An Important Example

### Traveling Salesman Problem (TSP)

- Very famous problem
- Very important problem
- Very easy to state/describe
- Very difficult to solve

### Informal Statement

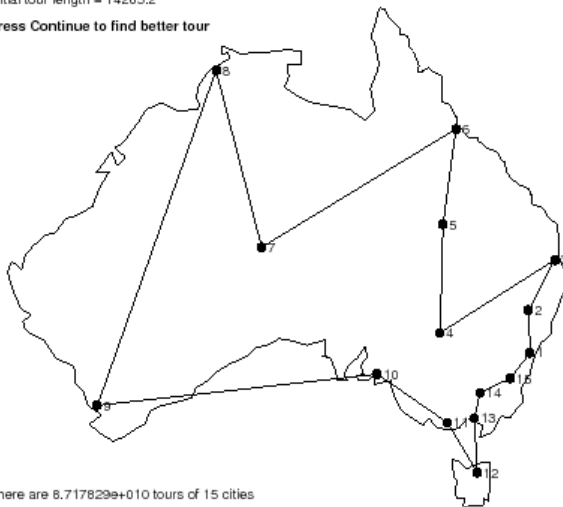
- **Given:**  
A set of cities and the direct distances between them.
- **Task:**  
Find the **shortest** closed route connecting these cities.

Note: the mathematical model of the TSP describes numerous other decision-problems that have nothing to do with the TSP.

# TSP – 15 cities

Initial tour length = 14265.2

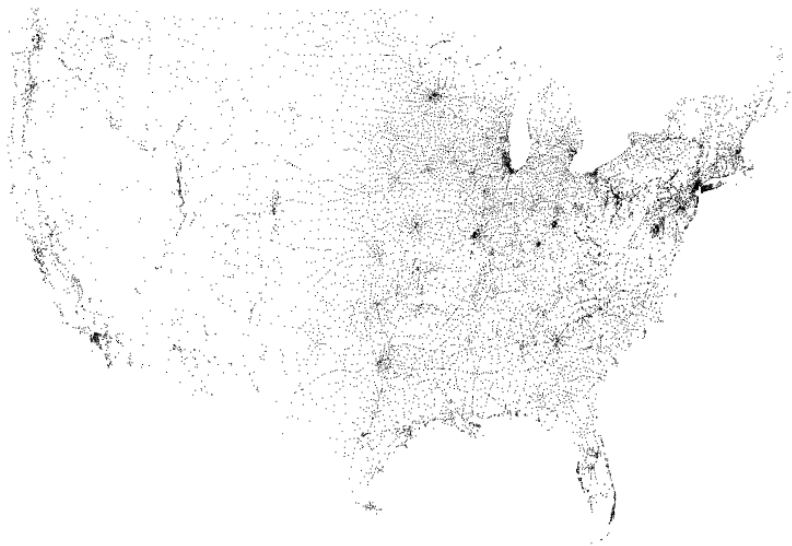
Press Continue to find better tour



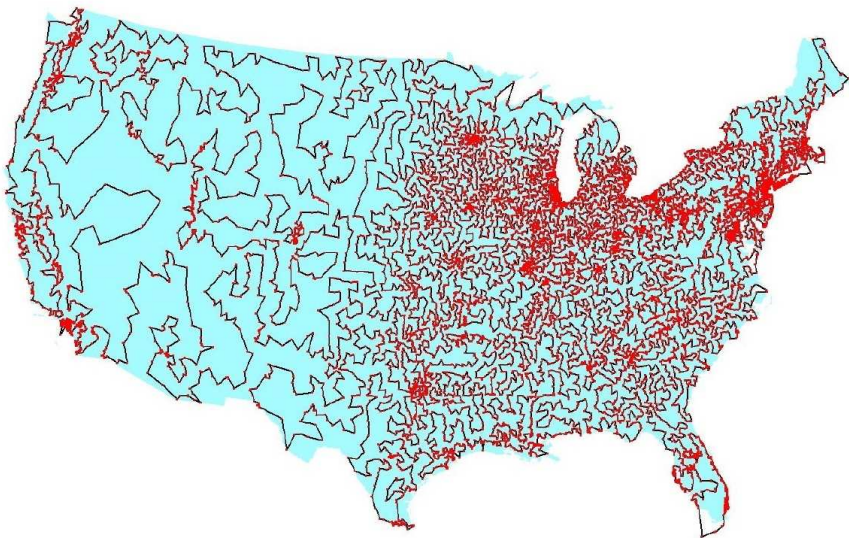
There are  $8.717829 \times 10^{10}$  tours of 15 cities

[http://www.parabola.unsw.edu.au/vol37\\_no2/node1.html](http://www.parabola.unsw.edu.au/vol37_no2/node1.html)

# TSP – 13,509 cities



# TSP – 13,509 cities



<http://www.cs.princeton.edu/courses/archive/spr01/cs126/checklist/tsp13509-sol.jpg>

# The Art of TSP



<http://www.oberlin.edu/math/faculty/bosch/tspart-page.html>

# The Art of TSP



£14.99

“Created using TSP  
computer programming”



[http://www.topbananagifts.co.uk/index.php?main\\_page=product\\_info  
&products\\_id=353](http://www.topbananagifts.co.uk/index.php?main_page=product_info&products_id=353)



# The Art of TSP



# The Art of TSP — 100,000 cities



<http://www.tsp.gatech.edu/data/ml/monalisa.html>

## Math-inside

### Timeout # 2

Why should we use mathematical models? What is the advantage/limitations of math models?

There are other types of models, e.g.:

- Graphic models
- Analog models
- Scale models

## About Models ...



## About Models ...



## About Models ...



## About Models ...



## About Models ...





## A simple math model

### Euclid, Elements, II.4, 300 B.C

If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.

---

## A simple math model

### Euclid, Elements, II.4, 300 B.C

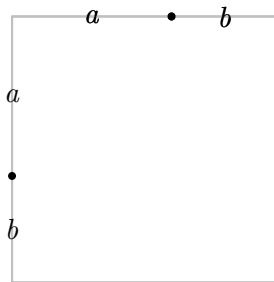
If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.



## A simple math model

### Euclid, Elements, II.4, 300 B.C

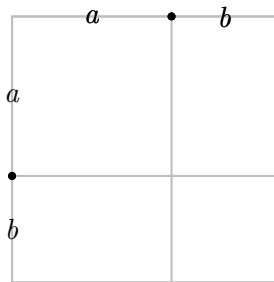
If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.



## A simple math model

### Euclid, Elements, II.4, 300 B.C

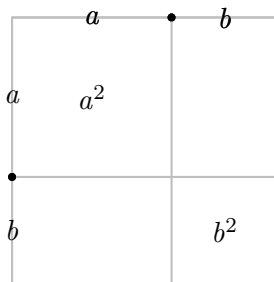
If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.



## A simple math model

### Euclid, Elements, II.4, 300 B.C

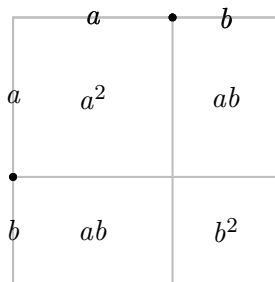
If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.



## A simple math model

### Euclid, Elements, II.4, 300 B.C

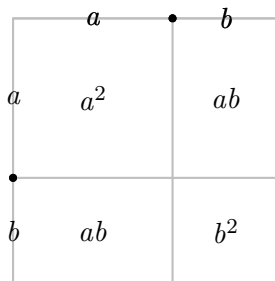
If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.



## A simple math model

### Euclid, Elements, II.4, 300 B.C

If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.



$$(a + b)^2 = a^2 + b^2 + 2ab$$

## Back to the TSP

### Plain English Model

If a salesman, starting from his home city, is to visit exactly once each city on a given list and then return home, it is plausible for him to select the order in which he visits the cities so that the total of the distances travelled in his tour is as small as possible.

[http://www.parabola.unsw.edu.au/vol37\\_no2/node1.html](http://www.parabola.unsw.edu.au/vol37_no2/node1.html)

### Math Model

$$\begin{aligned} \min_{x_1, \dots, x_n} & \left\{ \sum_{j=1}^{n-1} d(x_j, x_{j+1}) + d(x_n, x_1) \right\} \\ \text{subject to} & \{x_1, \dots, x_n\} = \{1, 2, \dots, n\} \end{aligned}$$

There are  $n$  **unknowns**:  $x_1, x_2, \dots, x_n$ , where  
 $x_j := j$ -th city on the tour,  $j = 1, 2, \dots, n$



## Math-inside

### Timeout # 3

Note the distinction between

- Mathematical **models** used to **describe** real-world problems.
- Mathematical **methods** used to **analyze/solve** the math problems defined by these models.

# TSP

## Math Model

$$\min_{x_1, \dots, x_n} \left\{ \sum_{j=1}^{n-1} d(x_j, x_{j+1}) + d(x_n, x_1) \right\}$$

subject to  $\{x_1, \dots, x_n\} = \{1, 2, \dots, n\}$

## Question

How do we **solve** the generic math problem defined by this model?

How do we **determine the (best) values** of the  $n$  unknowns  $x_1, \dots, x_n$ ?

## Answer

Using mathematical **methods** (algorithms) that were designed for **this** purpose.

## Back to decision-making

### Bad news

- The mathematical models/methods are problem-oriented: different problems requires different models/methods.
- No single, foolproof math-recipe can do the job.
- Most of the models are not “accessible” to “mathematically naive” users, let alone the general public.

### Good news

- The mathematical models/methods are problem-oriented: different problems requires different models/methods.
- No single, foolproof math-recipe can do the job.
- Most of the models are not “accessible” to “mathematically naive” users, let alone the general public.

Time ?

## Challenges and opportunities

Recall that (quantitative) decision-making models consist of the following ingredients:

- Decisions variables (the “unknowns”)
- Objectives and goals
- Requirements and constraints
- Preference structure
- Parameters

### TSP

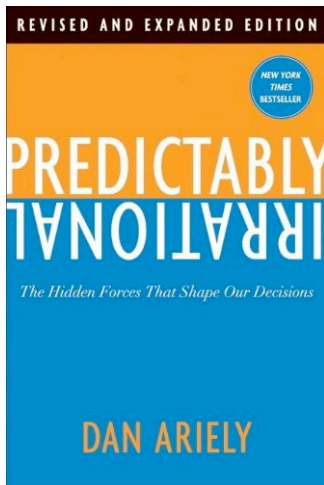
$$\begin{aligned} & \min_{x_1, \dots, x_n} \left\{ \sum_{j=1}^{n-1} d(x_j, x_{j+1}) + d(x_n, x_1) \right\} \\ & \text{subject to } \{x_1, \dots, x_n\} = \{1, 2, \dots, n\} \end{aligned}$$

## Challenges and opportunities

Just to mention a few these (arbitrary order):

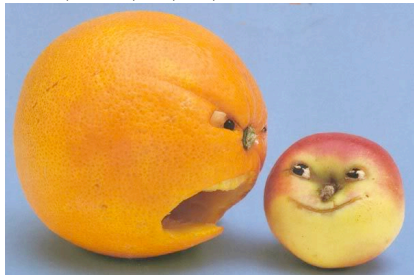
- Rationality and human factors
- Multi-objective
- Severe uncertainty
- Curse of dimensionality
- Heuristics
- PR

# Rationality and Human Factors



## Multi-objective

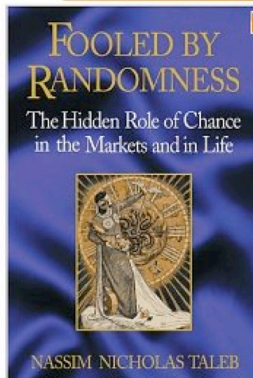
<http://ynevar.wordpress.com/2010/04/13/diversity-training-apples-vs-oranges-pt-ii/>



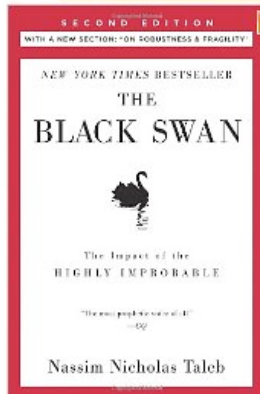


# Severe Uncertainty

Click to **LOOK INSIDE!**



Click to **LOOK INSIDE!**



# Curse of Dimensionality



VS

$n!$

$2^n$

# Heuristics

<http://www.hist.msu.ru/Labs/HisLab/Stud/Heuristics/english.htm>



*"Go ask your search engine."*

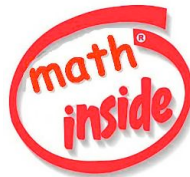
©The New Yorker, February 7, 2000

## PR



£14.99

“Created using TSP  
computer programming”

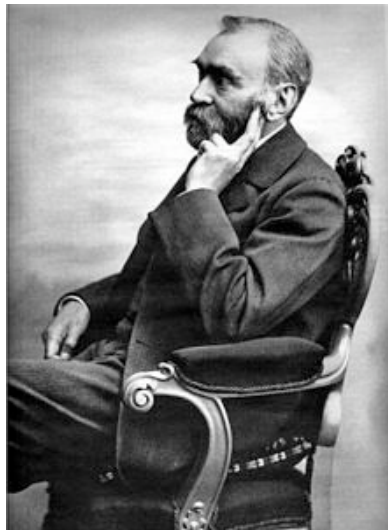


[http://www.topbananagifts.co.uk/index.php?main\\_page=product\\_info  
&products\\_id=353](http://www.topbananagifts.co.uk/index.php?main_page=product_info&products_id=353)

# Nobel Opportunities

**Alfred Bernhard Nobel**  
(1833 - 1896)

Swedish chemist, engineer,  
innovator, and armaments  
manufacturer



# Nobel Opportunities



There is no Nobel Prize for **Mathematics**  
There is no Nobel Prize for **Decision-Making**

## Nobel Opportunities

### Remark

The following is a sample of Nobel Prizes awarded to individuals whose work had a significant **math** content and a significant contribution in the area of **decision-making**.

# Economic Sciences 2011



Photo: NYU Stern

Thomas J. Sargent

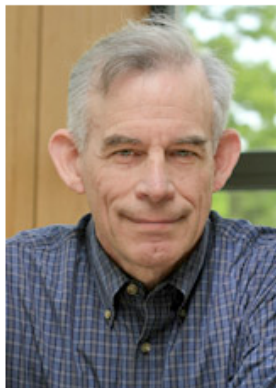


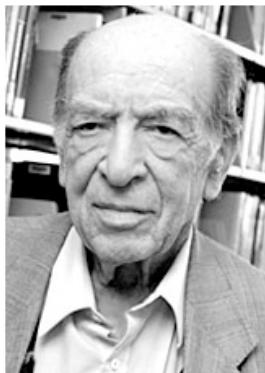
Photo: Denise Applewhite, Princeton University

Christopher A. Sims

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2011 was awarded jointly to Thomas J. Sargent and Christopher A. Sims *"for their empirical research on cause and effect in the macroeconomy"*



## Economic Sciences 2007



© University of Minnesota Photo: E. Ayoubzadeh

**Leonid Hurwicz**



© The Nobel Foundation Photo: U. Montan

**Eric S. Maskin**



© The Nobel Foundation Photo: U. Montan

**Roger B. Myerson**

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2007 was awarded jointly to Leonid Hurwicz, Eric S. Maskin and Roger B. Myerson *"for having laid the foundations of mechanism design theory"*.

## Economic Sciences 2005



Photo: D. Porges

**Robert J. Aumann**

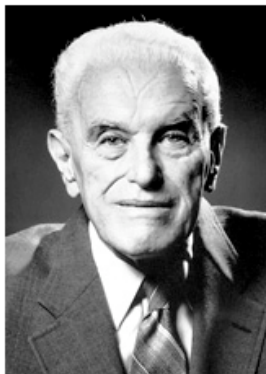


Photo: T. Zadig

**Thomas C. Schelling**

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2005 was awarded jointly to Robert J. Aumann and Thomas C. Schelling *"for having enhanced our understanding of conflict and cooperation through game-theory analysis"*

# Economic Sciences 1994



John C. Harsanyi



John F. Nash Jr.



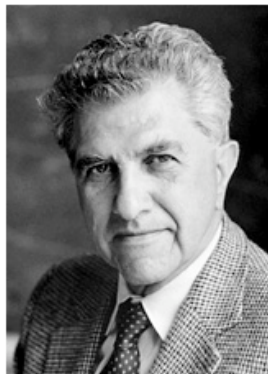
Reinhard Selten

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1994 was awarded jointly to John C. Harsanyi, John F. Nash Jr. and Reinhard Selten *"for their pioneering analysis of equilibria in the theory of non-cooperative games"*.

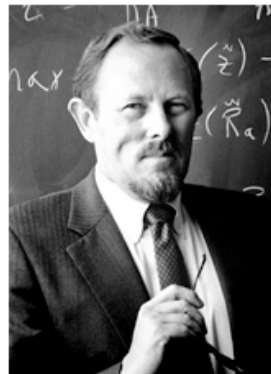
# Economic Sciences 1990



Harry M. Markowitz



Merton H. Miller



William F. Sharpe

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1990 was awarded jointly to Harry M. Markowitz, Merton H. Miller and William F. Sharpe "for their pioneering work in the theory of financial economics".

# Economic Sciences 1988



**Maurice Allais**

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1988 was awarded to Maurice Allais *"for his pioneering contributions to the theory of markets and efficient utilization of resources"*.

# Economic Sciences 1975



Leonid Vitaliyevich  
Kantorovich



Tjalling C. Koopmans

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1975 was awarded jointly to Leonid Vitaliyevich Kantorovich and Tjalling C. Koopmans *"for their contributions to the theory of optimum allocation of resources"*

# Economic Sciences 1972



John R. Hicks



Kenneth J. Arrow

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1972 was awarded jointly to John R. Hicks and Kenneth J. Arrow *"for their pioneering contributions to general economic equilibrium theory and welfare theory"*

# There is something about Game Theory





# Game Theory

## Two-Player Zero-Sum Game (circa 1928)




	$D_1$	$D_2$	$D_3$	$D_4$
$d_1$	1	2	3	4
$d_2$	6	3	4	5
$d_3$	7	1	8	2

Entries: Payoff to Row Player = Cost to Column player.

What is the best decision for each player?

Major difficulty: **Uncertainty**

**Remedy?**

## In the stable ...



## Saddle (equilibrium) point



# Game Theory

## Saddle (Equilibrium) Point




	$D_1$	$D_2$	$D_3$	$D_4$
$d_1$	1	2	3	4
$d_2$	6	3	4	5
$d_3$	7	1	8	2

Smallest entry in its row and largest entry in its column.

Difficulty: What happens in cases where there are no saddle points?

# Game Theory

## Saddle (Equilibrium) Point

Google



	$D_1$	$D_2$	$D_3$	$D_4$
$d_1$	1	2	3	4
$d_2$	6	3	4	5
$d_3$	7	4	8	2

There are no saddle points!

Remedy

Mixed Strategies:



# Game Theory

## The Prisoner's Dilemma

		Prisoner B	
		Silent	Sing
Prisoner A	Silent	1 , 1	5 , 0
	Sing	0 , 5	3 , 3

Entries: years in jail.

Help! (3,3) is an equilibrium point, but ... it is much worse than (1,1). Unfortunately, (1,1) is not an equilibrium point!

## How about **social** games?

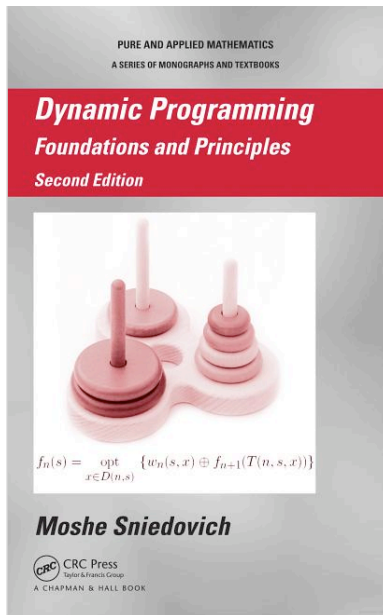
	4			6	9		1	
1					5			
	9		7			8		5
		4			8		7	
	3						6	
	6		2			1		
8		9			4		5	
			5					2
	7		1	8			3	

## How about **social** games?





# How about **social** games?



## Summary

- The **unreasonable effectiveness of mathematics** phenomenon applies to the area of decision-making.
- Indeed, mathematics plays a **central role** in decision-theory and decision-analysis.
- There are many ... **challenges and opportunities** in this business!
- Reveal the Math in your products



- Beware of **voodoo** decision theories (see my website).
- **Keywords:** Decision-theory, decision-analysis, operations research, operations analysis, optimization, analytics.

## Summary

And ... don't forget:

*A Theorem a day  
Keeps the doctor away!*

*Thank  
You*

